# Kinetic Quantum Theory of Gravity 

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#### Abstract

Starting from the action function we have derived a theoretical background that leads to quantization of gravity and the deduction of a correlation between the gravitational and inertial masses, which depends on the kinetic momentum of the particle. We show that there is a reaffirmation of the strong equivalence principle and consequently the Einstein's equations are preserved. In fact such equations are deduced here directly from this kinetic approach to Gravity. Moreover, we have obtained a generalized equation for inertial forces, which incorporates the Mach's principle into Gravitation. Also, we have deduced the equation of Entropy; the Hamiltonian for a particle in an electromagnetic field and the reciprocal fine structure constant. It is possible to deduce the expression of the Casimir force and also to explain the Inflation Period and the Missing Matter without assuming the existence of vacuum fluctuations. This new approach for Gravity will allow us to understand some crucial matters in Cosmology.


## 1. INTRODUCTION

Quantum Gravity was originally studied, by Dirac and others, as the problem of quantizing General Relativity. This approach has many difficulties, detailed by Isham [1]. In the 1970's physicists tried an even more conventional approach: simplify the Einstein's equations by pretending that they are almost linear, and then apply the standard methods of quantum field theory to the thusoversimplified equations. But this method, too, failed. In the 1980's a very different approach, known as string theory, became popular. For a while there are many enthusiasts of string theory. But the mathematical difficulties in string theory are formidable, and it is far from clear that they will be resolved any time soon. At the end of 1997 Isham [2] pointed out several "Structural Problems Facing Quantum Gravity Theory". At the beginning of this new century, the problem of quantizing the gravitational field was still open.

In this work we propose a new approach to Quantum Gravity where gravity is quantized starting from the generalization of the action function for a particle. The Einstein's equations of the General Relativity are deduced directly from this theory of Quantum Gravity. Also, it leads to a complete description of the Electromagnetic Field, providing a consistent unification of gravity with electromagnetism.

## 2. THEORY

We start with the action for a freeparticle that, as we know, is given by:

$$
S=-\alpha \int_{a}^{b} d s
$$

where $\quad \alpha$ is a quantity which characterize the particle.

In Relativistic Mechanics, the action can be written in the following form [3]:

$$
S=\int_{t_{1}}^{t_{2}} L d t=-\int_{t_{1}}^{t_{2}} \alpha c \sqrt{1-V^{2} / c^{2}} d t
$$

where

$$
L=-\alpha c \sqrt{1-V^{2} / c^{2}}
$$

is the Lagrange's function.
In Classical Mechanics the Lagrange's function for a free-particle is, as we know, given by: $L=a V^{2}$ where $V$ is the speed of the particle and $a$ a quantity hypothetically [4] given by :

$$
a=m / 2
$$

where $m$ is the mass of the particle. However, there is no distinction about the kind of mass (if gravitational mass, $m_{g}$, or inertial mass $m_{i}$ ) neither about its sign $( \pm)$.

The correlation between $a$ and $\alpha$ can be established based on the fact that on the limit $c \rightarrow \infty$ the relativistic expression for $L$ must be reduced to the classic expression $L=a V^{2}$. The result [5] is : $L=\alpha V^{2} / 2 c$. Therefore, if $\alpha=2 a c=m c \quad$ we obtain $L=a V^{2}$. Now, we must decide if $m=m_{g}$ or
$m=m_{i}$. We will see in this work that the definition of $m_{g}$ includes $m_{i}$. Thus the right option is $m_{g}$, i.e.,

$$
a=m_{g} / 2 .
$$

Consequently, $\alpha=m_{g} c \quad$ and the generalized expression for the action for a free-particle will have the following form:

$$
\begin{equation*}
S=-m_{g} c \int_{a}^{b} d s \tag{1}
\end{equation*}
$$

or

$$
\begin{equation*}
S=-\int_{t_{1}}^{t_{2}} m_{g} c^{2} \sqrt{1-V^{2} / c^{2}} d t \tag{2}
\end{equation*}
$$

where the Lagrange's function is

$$
\begin{equation*}
L=-m_{g} c^{2} \sqrt{1-V^{2} / c^{2}} \tag{3}
\end{equation*}
$$

The integral $S=\int_{t_{1}}^{t_{2}} m_{g} c^{2} \sqrt{1-V^{2} / c^{2}} d t$ preceded by the plus sign cannot have a minimum. Thus, the integrand of Eq.(2) must be always positive. Therefore, if $m_{g}>0$ then necessarily $t>0$; if $m_{g}<0$ then $t<0$. The possibility of $t<0$ is based on the well-known equation $t= \pm t_{0} / \sqrt{1-V^{2} / c^{2}}$ of Einstein's Theory.

Thus if the gravitational mass of a particle is positive then $t$ is also positive and therefore given by $t=+t_{0} / \sqrt{1-V^{2} / c^{2}}$. This leads to the well-known relativistic prediction that the particle goes to the future if $V \rightarrow c$. However, if the gravitational mass of the particle is negative then $t$ is negative and given by $t=-t_{0} / \sqrt{1-V^{2} / c^{2}}$. In this case the prediction is that the particle goes to the past if $V \rightarrow c$. Consequently, $m_{g}<0$ is the necessary condition for the particle to go to the past. Further on it will be derived a correlation between gravitational and inertial masses, which contains the possibility of $m_{g}<0$.

The Lorentz's transforms follow the same rule for $m_{g}>0$ and $m_{g}<0$, i.e., the sign before $\sqrt{1-V^{2} / c^{2}}$ will be (+) when $m_{g}>0$ and (-) if $m_{g}<0$.

The momentum, as we know, is the vector $\vec{p}=\partial L / \partial \vec{V}$.Thus from Eq.(3) we obtain

$$
\begin{equation*}
\vec{p}=\frac{m_{g} \vec{V}}{ \pm \sqrt{1-V^{2} / c^{2}}}=M_{g} \vec{V} \tag{4}
\end{equation*}
$$

The sign $(+)$ in the equation above will be used when $m_{g}>0$ and the $\operatorname{sign}(-)$ if $m_{g}<0$. Henceforth, by simplicity the signs $( \pm)$ before $\sqrt{1-V^{2} / c^{2}}$ will be omitted.
The derivate $d \vec{p} / d t$ is the inertial force $F_{i}$ which acts on the particle. If the force is perpendicular to the speed we have

$$
\begin{equation*}
\vec{F}_{i}=\frac{m_{g}}{\sqrt{1-V^{2} / c^{2}}} \frac{d \vec{V}}{d t} \tag{5}
\end{equation*}
$$

However, if the force and the speed have the same direction, we find that

$$
\begin{equation*}
\vec{F}_{i}=\frac{m_{g}}{\left(1-V^{2} / c^{2}\right)^{3 / 2}} \frac{d \vec{V}}{d t} \tag{6}
\end{equation*}
$$

From Mechanics [6] we know that $\vec{p} \cdot \vec{V}-L$ denotes the energy of the particle, thus we can write

$$
\begin{equation*}
E_{g}=\vec{p} \cdot \vec{V}-L=\frac{m_{g} c^{2}}{\sqrt{1-V^{2} / c^{2}}}=M_{g} c^{2} \tag{7}
\end{equation*}
$$

This fundamental equation presents the concept of Gravitational Energy, $\quad E_{g}$, in addition to the well-known concept of Inertial Energy, $E_{i}$, and shows that $E_{g}$ is not null for $V=0$, but it has the finite value

$$
\begin{equation*}
E_{g 0}=m_{g} c^{2} \tag{8}
\end{equation*}
$$

This is the particle's gravitational energy at rest.

The Eq.(7) can be rewritten in the following form:

$$
\begin{align*}
E_{g} & =m_{g} c^{2}-\frac{m_{g} c^{2}}{\sqrt{1-V^{2} / c^{2}}}-m_{g} c^{2}= \\
& =\frac{m_{g}}{m_{i}}[m_{i} c^{2}+\underbrace{\left(\frac{m_{i} c^{2}}{\sqrt{1-V^{2} / c^{2}}}-m_{i} c^{2}\right)}_{E_{K i}}]= \\
& =\frac{m_{g}}{m_{i}}\left(E_{i 0}+E_{K i}\right)=\frac{m_{g}}{m_{i}} E_{i} \tag{9}
\end{align*}
$$

By analogy to the Eq.(8), $E_{i 0}=m_{i} c^{2}$ into the equation above, is the inertial energy at rest. Thus, $E_{i}=E_{i 0}+E_{K i}$ is the
total inertial energy, where $E_{K i}$ is the kinetic inertial energy. From the Eqs.(7) and (9) we thus obtain

$$
\begin{equation*}
E_{i}=\frac{m_{i} c^{2}}{\sqrt{1-V^{2} / c^{2}}}=M_{i} c^{2} \tag{10}
\end{equation*}
$$

For small velocities $(V \ll c)$, we obtain

$$
\begin{equation*}
E_{i} \approx m_{i} c^{2}+\frac{1}{2} m_{i} V^{2} \tag{11}
\end{equation*}
$$

where we recognize the classical expression for the kinetic inertial energy of the particle.

The expression for the kinetic gravitational energy, $E_{K g}$, is easily deduced by comparing of the Eqs.(7) and (9). The result is

$$
\begin{equation*}
E_{K g}=\frac{m_{g}}{m_{i}} E_{K i} . \tag{12}
\end{equation*}
$$

In the presented picture, we can say that the gravity, $\vec{g}$, into a gravitational field produced by a particle of gravitational mass $m_{g}$ depends on the particle's gravitational energy, $E_{g}$ ( given by Eq.(7)), because we can write
$g=-G \frac{E_{g}}{r^{2} c^{2}}=-G \frac{M_{g} c^{2}}{r^{2} c^{2}}=-G \frac{M_{g}}{r^{2}}$
where $\quad M_{g}=m_{g}\left(1-V^{2} / c^{2}\right)^{-1 / 2}$ is the relativistic gravitational mass defined in the Eqs.(4)and (7).

On the other hand, as we know, the gravitational force is conservative. Thus, gravitational energy, in agreement with the energy conservation law, can be expressed by the decrease of the inertial energy, i.e.,

$$
\begin{equation*}
\Delta E_{g}=-\Delta E_{i} \tag{14}
\end{equation*}
$$

This equation expresses the fact that the decrease of gravitational energy corresponds to an increase of the inertial energy.

Therefore a variation $\Delta E_{i}$ in $E_{i}$ yields a variation $\Delta E_{g}=-\Delta E_{i}$ in $E_{g}$. Thus $E_{i}=E_{i 0}+\Delta E_{i} ; E_{g}=E_{g 0}+\Delta E_{g}=E_{g 0}-\Delta E_{i} \quad$ and

$$
\begin{equation*}
E_{g}+E_{i}=E_{g 0}+E_{i 0} \tag{15}
\end{equation*}
$$

Comparison between (7)and(10) shows that $E_{g 0}=E_{i 0}$. Consequently we have

$$
\begin{equation*}
E_{g}+E_{i}=E_{g 0}+E_{i 0}=2 E_{i 0} \tag{16}
\end{equation*}
$$

However, $E_{i}=E_{i 0}+E_{K i}$. Thus (16) becomes

$$
\begin{equation*}
E_{g}=E_{i 0}-E_{K i} . \tag{17}
\end{equation*}
$$

Note the symmetry in the equations of $E_{i}$ and $E_{g}$. Substitution of $E_{i 0}=E_{i}-E_{K i}$ into (17) yields

$$
\begin{equation*}
E_{i}-E_{g}=2 E_{K i} \tag{18}
\end{equation*}
$$

Squaring the Eqs.(4) and (7) and comparing the result, we find the following correlation between gravitational energy and momentum :

$$
\begin{equation*}
\frac{E_{g}^{2}}{c^{2}}=p^{2}+m_{g}{ }^{2} c^{2} . \tag{19}
\end{equation*}
$$

The energy expressed as a function of the momentum is, as we know, called Hamiltonian or Hamilton's function:

$$
\begin{equation*}
H_{g}=c \sqrt{p^{2}+m_{g}{ }^{2} c^{2}} \tag{20}
\end{equation*}
$$

It is known that starting from the Schrödinger equation we may obtain the well-known expression for energy of a particle in periodic motion inside a cubical box of edge length $L$ [ 7 ]. The result now is

$$
\begin{equation*}
E_{n}=\frac{n^{2} h^{2}}{8 m_{g} L^{2}} \quad n=1,2,3, \ldots \tag{21}
\end{equation*}
$$

Note that the term $h^{2} / 8 m_{g} L^{2}$ (energy) will be minimum for $L=L_{\max }$ where $L_{\max }$ is the maximum edge length of a cubical box whose maximum diameter

$$
\begin{equation*}
d_{\max }=L_{\max } \sqrt{3} \tag{22}
\end{equation*}
$$

is equal to the maximum "diameter" of the Universe.

The minimum energy of a particle is obviously its inertial energy at rest $m_{g} c^{2}=m_{i} c^{2}$. Therefore we can write

$$
\frac{n^{2} h^{2}}{8 m_{g} L_{\max }^{2}}=m_{g} c^{2}
$$

Then from the equation above follows that

$$
\begin{equation*}
m_{g}= \pm \frac{n h}{c L_{\max } \sqrt{8}} \tag{23}
\end{equation*}
$$

whence we see that there is a minimum value for $m_{g}$ given by

$$
\begin{equation*}
m_{g(\min )}= \pm \frac{h}{c L_{\max } \sqrt{8}} \tag{24}
\end{equation*}
$$

The relativistic gravitational mass
$M_{g}=m_{g}\left(1-V^{2} / c^{2}\right)^{-1 / 2}$, defined in the Eqs.(4) and (7), shows that

$$
\begin{equation*}
M_{g(\text { min })}=m_{g(\text { min })} \tag{25}
\end{equation*}
$$

The box normalization leads to conclusion that the propagation number $k=|\vec{k}|=2 \pi / \lambda$ is restricted to the values $k=2 \pi n / L$. This is deduced assuming an arbitrarily large but finite cubical box of volume $L^{3}$ [8]. Thus we have

$$
L=n \lambda
$$

From this equation we conclude that

$$
n_{\text {max }}=\frac{L_{\text {max }}}{\lambda_{\text {min }}}
$$

and

$$
L_{\text {min }}=n_{\text {min }} \lambda_{\text {min }}=\lambda_{\text {min }}
$$

Since $n_{\text {min }}=1$. Therefore we can write that

$$
\begin{equation*}
L_{\text {max }}=n_{\text {max }} L_{\text {min }} \tag{26}
\end{equation*}
$$

From this equation we thus conclude that

$$
\begin{equation*}
L=n L_{\text {min }} \tag{27}
\end{equation*}
$$

or

$$
\begin{equation*}
L=\frac{L_{\text {max }}}{n} \tag{28}
\end{equation*}
$$

Multiplying (27) and (28) by $\sqrt{3}$ and reminding that $d=L \sqrt{3}$, we obtain

$$
\begin{equation*}
d=n d_{\min } \quad \text { or } \quad d=\frac{d_{\max }}{n} \tag{29}
\end{equation*}
$$

Equations above show that the length (and therefore the space) is quantized.

By analogy to (23) we can also conclude that

$$
\begin{equation*}
M_{g(\max )}= \pm \frac{n_{\max } h}{c L_{\min } \sqrt{8}} \tag{30}
\end{equation*}
$$

since the relativistic gravitational mass, $M_{g}=m_{g}\left(1-V^{2} / c^{2}\right)^{-1 / 2}$, is just multiple of $m_{g}$.

Equation (26) tells us that $L_{\text {min }}=L_{\text {max }} / n_{\text {max }}$. Thus Eq.(30) can be written as follows

$$
\begin{equation*}
M_{g(\max )}= \pm \frac{n_{\text {max }}^{2} h}{c L_{\text {max }} \sqrt{8}} \tag{31}
\end{equation*}
$$

Comparison of (31) with (24) shows that

$$
\begin{equation*}
M_{g(\text { max })}=n_{\text {max }}^{2} m_{g(\text { min })} \tag{32}
\end{equation*}
$$

which leads to following conclusion that

$$
\begin{equation*}
M_{g}=n^{2} m_{g(\min )} \tag{33}
\end{equation*}
$$

This equation shows that the gravitational mass is quantized.

Substitution of (33) into (13) leads to quantization of gravity, i.e.,

$$
\begin{align*}
g & =-\frac{G M_{g}}{r^{2}}=n^{2}\left(-\frac{G m_{g(\min )}}{\left(r_{\max } / n\right)^{2}}\right)= \\
& =n^{4} g_{\text {min }} \tag{34}
\end{align*}
$$

From the Hubble's law follows that

$$
\begin{aligned}
& V_{\max }=\tilde{H} l_{\text {max }}=\tilde{H}\left(d_{\max } / 2\right) \\
& V_{\text {min }}=\tilde{H} l_{\text {min }}=\tilde{H}\left(d_{\text {min }} / 2\right)
\end{aligned}
$$

whence

$$
\frac{V_{\text {max }}}{V_{\text {min }}}=\frac{d_{\text {max }}}{d_{\text {min }}}
$$

Equations (29) tell us that $d_{\text {max }} / d_{\text {min }}=n_{\text {max }}$. Thus the equation above gives

$$
\begin{equation*}
V_{\text {min }}=\frac{V_{\text {max }}}{n_{\text {max }}} \tag{35}
\end{equation*}
$$

which leads to following conclusion

$$
V=\frac{V_{\text {max }}}{n}
$$

this equation shows that velocity is also quantized.

From this equation one concludes that we can have $V=V_{\text {max }}$ or $V=V_{\text {max }} / 2$, but nothing in between. This shows clearly that $V_{\text {max }}$ cannot be equal to $c$ (speed of light in vacuum). Thus follows that

$$
\begin{aligned}
& n=1 \quad V=V_{\text {max }} \\
& n=2 \quad V=V_{\text {max }} / 2 \\
& n=3 \quad V=V_{\text {max }} / 3 \\
& \text { Tachyons } \\
& n=n_{x}-1 \quad V=V_{\text {max }} /\left(n_{x}-1\right) \\
& n=n_{x} \quad V=V_{\text {max }} / n_{x}=c \leftarrow \\
& n=n_{x}+1 \quad V=V_{\max } /\left(n_{x}+1\right) \quad \text { Tardions } \\
& n=n_{x}+2 \quad V=V_{\max } /\left(n_{x}+2\right)
\end{aligned}
$$

where $n_{x}$ is a very big number.
Then $c$ is the upper limit of speed of the Tardions and also the lower limit of
speed of the Tachyons. Obviously that limit is always the same in all inertial frames. Therefore $c$ can be used like a reference speed, which we may compare any speed $V$, as occurs in the relativistic factor $\sqrt{1-V^{2} / c^{2}}$. Thus in this factor $c$ not refers to maximum propagation speed of the interactions such as suggest some authors; $c$ is just a speed limit which is the same in any inertial frame.

The temporal coordinate $x^{0}$ of the space-time is now $x^{0}=V_{\text {max }} t \quad\left(x^{0}=c t\right.$ is then obtained when $\left.\quad V_{\max } \rightarrow c\right)$. Substitution of $V_{\max }=n V=n(\tilde{H l})$ into this equation yields $t=x^{0} / V_{\max }=(1 / n \tilde{H})\left(x^{0} / l\right)$. On the other hand, since $V=\tilde{H} l$ and $V=V_{\max } / n$ then we can write that $l=V_{\max } \tilde{H}^{-1} / n$. Thus $\left(x^{0} / l\right)=\tilde{H}(n t)=\tilde{H} t_{\max }$. Therefore we can finally write

$$
\begin{equation*}
t=(1 / n \tilde{H})\left(x^{0} / l\right)=t_{\max } / n \tag{37}
\end{equation*}
$$

which shows the quantization of time.
Now let us go back to Eq. (20) which will be called the gravitational Hamiltonian to distinguish it from the inertial Hamiltonian $H_{i}$ :

$$
\begin{equation*}
H_{i}=c \sqrt{p^{2}+m_{i}^{2} c^{2}} \tag{38}
\end{equation*}
$$

Consequently, the Eq. (18) can be rewritten in the following form:

$$
\begin{equation*}
H_{i}-H_{g}=2 \Delta H_{i} \tag{39}
\end{equation*}
$$

where $\Delta H_{i}$ is the variation on the inertial Hamiltonian or inertial kinetic energy. A momentum variation $\Delta p$ yields a variation $\Delta H_{i}$ given by:
$\Delta H_{i}=\sqrt{(p+4)^{2} c^{2}+m_{i}^{2} c^{4}}-\sqrt{p^{2} c^{2}+m_{i}^{2} c^{4}}$
Substituting Eqs.(20), (38) and (40) into (39) and making $p=0$, we obtain
$m_{g} c^{2}-m_{i} c^{2}=2\left(\sqrt{\Delta p^{2} c^{2}+m_{i}^{2} c^{4}}-m_{i} c^{2}\right)$ Fro
m this equation we derive the general expression of correlation between the gravitational and inertial mass, i.e.,

$$
\begin{equation*}
m_{g}=m_{i}-2\left[\sqrt{1+\left(\frac{\Delta p}{m_{i} c}\right)^{2}}-1\right] m_{i} \tag{41}
\end{equation*}
$$

Note that for $\Delta p>m_{i} c(\sqrt{5} / 2)$ the value of $m_{g}$ becomes negative.

Equation (41) can also be expressed in terms of velocity $V$ of the particle. Starting from (4) we can write

$$
(p+\Delta p)=\frac{\left(m_{g}-\Delta m_{g}\right)(V+\Delta V)}{\sqrt{1-(V+\Delta V)^{2} / c^{2}}}
$$

For $V=0 ; p=0$. Thus the equation above reduces to:
$\Delta p=\left(m_{g}-\Delta m_{g}\right) \Delta V / \sqrt{1-(\Delta V / c)^{2}}$ From the Eq.(16) we obtain:
$E_{g}=2 E_{i 0}-E_{i}=2 E_{i 0}-\left(E_{i 0}+\Delta E_{i}\right)=E_{i 0}-\Delta E_{i}$ However, Eq.(14) tells us that $-\Delta E_{i}=\Delta E_{g}$; it leads to $E_{g}=E_{0}+\Delta E_{g}$ or $m_{g}=m_{i}+\Delta m_{g}$. Thus, in the expression of $\Delta p$ we can replace $\left(m_{g}-\Delta m_{g}\right)$ by $m_{i}$, i.e.,

$$
\Delta p=m_{i} \Delta V / \sqrt{1-(\Delta V / c)^{2}}
$$

We can therefore write

$$
\begin{equation*}
\frac{\Delta p}{m_{i} c}=\frac{V / c}{\sqrt{1-(V / c)^{2}}} \tag{42}
\end{equation*}
$$

By substitution of the expression above into Eq.(41) we thus obtain:

$$
\begin{equation*}
m_{g}=m_{i}-2\left[\left(1-V^{2} / c^{2}\right)^{-\frac{1}{2}}-1\right] m_{i} \tag{43}
\end{equation*}
$$

For $V=0$ the Eq.(43) gives

$$
m_{g}=m_{i}
$$

Therefore, in this case, the previously obtained quantized relation (33),

$$
\begin{align*}
M_{g}=n^{2} m_{g(\text { min })} & , \text { becomes } \\
m_{i} & =n^{2} m_{i(\text { min })} \tag{44}
\end{align*}
$$

which shows the quantization of inertial mass.

Finally, by dividing both members of Eq.(43)by $\sqrt{1-V^{2} / c^{2}}$ we readily obtain

$$
\begin{equation*}
M_{g}=M_{i}-2\left[\left(1-V^{2} / c^{2}\right)^{-\frac{1}{2}}-1\right] M_{i} \tag{45}
\end{equation*}
$$

The Lorentz's force is usually written in the following form:

$$
d \vec{P} / d t=q \vec{E}+q \vec{V} \times \vec{B}
$$

where $\vec{P}=m_{i} \vec{V} / \sqrt{1-V^{2} / c^{2}}$. However, Eq.(4) tell us that $\vec{p}=m_{g} V / \sqrt{1-V^{2} / c^{2}}$.

Therefore, the expressions above must be corrected by multiplying its members by $m_{g} / m_{i}$, i.e.,

$$
\vec{P} \frac{m_{g}}{m_{i}}=\frac{m_{g}}{m_{i}} \frac{m_{i} \vec{V}}{\sqrt{1-V^{2} / c^{2}}}=\frac{m_{s} \vec{V}}{\sqrt{1-V^{2} / c^{2}}}=\vec{p}
$$

and

$$
\frac{d \vec{p}}{d t}=\frac{d}{d t}\left(\vec{P} \frac{m_{g}}{m_{i}}\right)=(q \vec{E}+q \vec{V} \times \vec{B}) \frac{m_{g}}{m_{i}}
$$

That is now the general expression for Lorentz's force.

When the force is perpendicular to the speed, the Eq.(5) gives $d \vec{p} / d t=m_{g}(d \vec{V} / d t) / \sqrt{1-V^{2} / c^{2}}$. By comparing with Eq.(46) we thus obtain

$$
\left(m_{i} / \sqrt{1-V^{2} / c^{2}}\right)(d \vec{V} / d t)=q \vec{E}+q \vec{V} \times \vec{B}
$$

Starting from this equation, well-known experiments have been carried out in order to verify the relativistic expression: $m_{i} / \sqrt{1-V^{2} / c^{2}}$.

In particular, we can look on the momentum variation $(\Delta p)$ as due to absorption or emission of electromagnetic energy by the particle ( by means of radiation and/or by means of Lorentz's force upon the charge of the particle).

In the case of radiation (any type), $\Delta p$ can be obtained as follows. It is known that the radiation pressure , $d P$, upon an area $d A=d x d y$ of a volume $d V=d x d y d z$ of a particle ( the incident radiation normal to the surface $d A$ ) is equal to the energy $d U$ absorbed per unit volume $(d U / d V) \quad$ i.e.,

$$
\begin{equation*}
d P=\frac{d U}{d V}=\frac{d U}{d x d y d z}=\frac{d U}{d A d z} \tag{47}
\end{equation*}
$$

Substitution of $d z=v d t(v$ is the speed of radiation) into the equation above gives

$$
\begin{equation*}
d P=\frac{d U}{d V}=\frac{(d U / d A d t)}{v}=\frac{d D}{v} \tag{48}
\end{equation*}
$$

Since $d P d A=d F$ we can write:

$$
\begin{equation*}
d F d t=\frac{d U}{v} \tag{49}
\end{equation*}
$$

However we know that $d F=d p / d t$, then

$$
\begin{equation*}
d p=\frac{d U}{v} \tag{50}
\end{equation*}
$$

From Eq.(48) follows that

$$
\begin{equation*}
d U=d P d V=\frac{d V d D}{v} \tag{51}
\end{equation*}
$$

Substitution into (50) yields

$$
\begin{equation*}
d p=\frac{d V d D}{v^{2}} \tag{52}
\end{equation*}
$$

or

$$
\int_{0}^{\Delta p} d p=\frac{1}{v^{2}} \int_{0}^{D} \int_{0}^{V} d V d D
$$

whence

$$
\begin{equation*}
\Delta p=\frac{V D}{v^{2}} \tag{53}
\end{equation*}
$$

This expression is general for all types of waves. Including no-electromagnetic waves like sound waves. In this case, $v$ in Eq.(53), will be the speed of sound in the medium and $D$ the intensity of the sound radiation.

In the case of electromagnetic waves, the Electrodynamics tells us that $v$ will be given by

$$
v=\frac{d z}{d t}=\frac{\omega}{\kappa_{r}}=\frac{c}{\sqrt{\frac{\varepsilon_{r} \mu_{r}}{2}\left(\sqrt{1+(\sigma / \omega \varepsilon)^{2}}+1\right)}}
$$

Where $k_{r}$ is the real part of the propagation vector $\vec{k} ; k=|\vec{k}|=k_{r}+i k_{i} ; \varepsilon$, $\mu$ and $\sigma$, are the electromagnetic characteristics of the medium in which the incident (or emitted) radiation is propagating $\left(\varepsilon=\varepsilon_{r} \varepsilon_{0}\right.$ where $\varepsilon_{r}$ is the relative dielectric permittivity and $\varepsilon_{0}=8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m} ; \mu=\mu_{r} \mu_{0}$ where $\mu_{r}$ is the relative magnetic permeability and $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m} ; \quad \sigma$ is the electrical conductivity). For an atom inside a body, the incident(or emitted) radiation on this atom will be propagating inside the body, and consequently, $\sigma=\sigma_{\text {body }}, \varepsilon=\varepsilon_{\text {body }}, \mu=\mu_{\text {body }}$. It is then evident that the index of refraction $n_{r}=c / v$ will be given by
$n_{r}=\frac{c}{v}=\sqrt{\frac{\varepsilon_{r} \mu_{r}}{2}\left(\sqrt{1+(\sigma / \omega \varepsilon)^{2}}+1\right)}$
On the other hand, from Eq.(50) follows that

$$
\Delta p=\frac{U}{v}\left(\frac{c}{c}\right)=\frac{U}{c} n_{r}
$$

Substitution into Eq.(41) yields

$$
\begin{equation*}
m_{g}=\left\{1-2\left[\sqrt{1+\left(\frac{U}{m_{i} c^{2}} n_{r}\right)^{2}}-1\right]\right\} m_{i} \tag{55}
\end{equation*}
$$

For $\sigma \gg \omega \varepsilon$, the expression (54) gives

$$
\begin{equation*}
n_{r}=\frac{c}{v}=\sqrt{\frac{\mu \sigma c^{2}}{4 \pi f}} \tag{56}
\end{equation*}
$$

Substitution of (56) into (55) leads to

$$
m_{g}=\left\{1-2\left[\sqrt{1+\frac{\mu \sigma}{4 \pi f}\left(\frac{U}{m_{i} c}\right)^{2}}-1\right]\right\} m_{i}
$$

This equation shows that atoms of ferromagnetic materials with very-high $\mu$ can have its gravitational masses strongly reduced by means of Extremely Low Frequency (ELF) electromagnetic radiation. It also shows that atoms of superconducting materials (due to very-high $\sigma$ ) can also have its gravitational masses strongly reduced by means of ELF electromagnetic radiation.

Alternatively, we may put Eq.(55) as a function of the power density ( or intensity ), $D$, of the radiation. The integration of (51) gives $U=V D / v$. Thus we can write (55) in the following form:

$$
\begin{equation*}
m_{g}=\left\{1-2\left[\sqrt{1+\left(\frac{n_{r}^{2} D}{\rho c^{3}}\right)^{2}}-1\right]\right\} m_{i} \tag{57}
\end{equation*}
$$

where $\rho=m_{i} / V$.
For $\sigma \gg \omega \varepsilon, n_{r}$ will be given by (56) and consequently (57) becomes

$$
\begin{equation*}
m_{g}=\left\{1-2\left[\sqrt{1+\left(\frac{\mu \sigma D}{4 \pi f \rho c}\right)^{2}}-1\right]\right\} m_{i} \tag{58}
\end{equation*}
$$

The vector $\vec{D}=(U / V) \vec{v}$, which we may define from (48), has the same direction of the propagation vector $\vec{k}$ and evidently corresponds to the Poynting vector. Then $\vec{D}$ can be replaced by $\vec{E} \times \vec{H}$. Thus we can write $D=E H=E(B / \mu)=E[(E / v) / \mu]=(1 / v \mu) E^{2}$.
For $\sigma \gg \omega \varepsilon$ the Eq.(54) tells us that $v=\sqrt{4 \pi f / \mu \sigma}$ consequently we obtain

$$
D=E^{2} \sqrt{\frac{\sigma}{4 \pi f \mu}}
$$

This expression refers to the instantaneous values of $D$ and $E$. The average value for $E^{2}$ is equal to $1 / 2 E_{m}^{2}$ because $E$ varies sinusoidaly ( $E_{m}$ is the maximum value for $E$ ). Consequently equation above tells us that the average density $\bar{D}$ is given by

$$
\bar{D}=1 / 2 E_{m}^{2} \sqrt{\frac{\sigma}{4 \pi f \mu}}
$$

Substitution of this expression into (58) yields the expression for $\bar{m}_{g}$. Substitution of the expression of $D$ into (58) gives
$m_{g}=\left\{1-2\left[\sqrt{1+\mu\left(E^{2} / c \rho\right)^{2}(\sigma / 4 \pi f)^{3}}-1\right]\right\} m_{i}$
Note that for extremely-low frequencies the value of $f^{-3}$ in this equation becomes highly expressive.

Now consider an electric current $i=i_{0} \sin 2 \pi f t$ through a conductor. Since the current density, $\vec{J}$, is expressed by $\vec{J}=d i / d \vec{S}=\sigma \vec{E}$, then we can write that $E=i / \sigma S=\left(i_{0} / \sigma S\right) \sin 2 \pi f t$. Substitution of this equation into (59) gives
$m_{g}=\left\{1-2\left[\sqrt{1+\left(i_{0}^{4} \mu / 64 \pi^{3} c^{2} \rho^{2} S^{4} f^{3} \sigma\right) \sin ^{4} 2 \pi f t}-1\right]\right\} m_{i}$

If the conductor is a supermalloy rod $(1 \times 1 \times 400 \mathrm{~mm})$ then $\mu_{r}=100,000$ (initial); $\rho=8770 \mathrm{~kg} / \mathrm{m}^{3} ; \sigma=1.6 \times 10^{6} \mathrm{~S} / \mathrm{m}$ and $S=1 \times 10^{-6} \mathrm{~m}^{2}$. Substitution of these value into equation above yields the following expression for the gravitational mass of the supermalloy rod:
$m_{g(s m)}=\left\{1-2\left[\sqrt{1+\left(5.71 \times 10^{-12} i_{0}^{4} / f^{3}\right) \sin ^{4} 2 \pi f t}-1\right] \xi_{i(s m)}\right.$

Some oscillators like the HP3325A ( Op. 002 High Voltage Output ) can generate sinusoidal voltages with
extremely-low frequencies down to $f=1 \times 10^{-6} \mathrm{~Hz}$ and amplitude up to 20 V ( into $50 \Omega$ load). The maximum output current is $0.08 A_{p p}$.

Thus, for $i_{0}=0.04 A \quad\left(0.08 A_{p p}\right)$ and $f<2.25 \times 10^{-6} \mathrm{~Hz}$ the equation above shows that the gravitational mass of the rod becomes negative at $2 \pi f t=\pi / 2$; for $f \cong 1.7 \times 10^{-6} \mathrm{~Hz} \quad$ at $t=1 / 4 f=1.47 \times 10^{5} s \cong 40.8 \mathrm{~h}$ it shows that $m_{g(s m)} \cong-m_{i(s m)}$.

Let us now return to the theory.
The equivalence between frames of non-inertial reference and gravitational fields presupposed $m_{g} \equiv m_{i}$ because the inertial forces was given by $\vec{F}_{i}=m_{i} \vec{a}$, while the equivalent gravitational forces, by $\vec{F}_{g}=m_{g} \vec{g}$. Thus, to satisfy the equivalence ( $\vec{a} \equiv \vec{g} \quad$ and $\vec{F}_{i} \equiv F_{g}$ ) it was necessary that $m_{g} \equiv m_{i}$. Now, the inertial force, $\vec{F}_{i}$, is given by Eq.(6), and from the Eq.(13) we can obtain the gravitational force, $\vec{F}_{g}$. Thus, $\vec{F}_{i} \equiv \vec{F}_{g}$ leads to

$$
\begin{aligned}
& \frac{m_{g}}{\left(1-V^{2} / c^{2}\right)^{\frac{2}{2}}} \vec{a} \equiv G \frac{m_{g}^{\prime}}{\left(r^{\prime} \sqrt{1-V^{2} / c^{2}}\right)^{2}} \frac{m_{g}}{\sqrt{1-V^{2} / c^{2}}} \equiv \\
& \equiv\left(G \frac{m_{g}^{\prime}}{r^{\prime 2}}\right) \frac{m_{g}}{\left(1-V^{2} / c^{2}\right)^{3 / 2}} \equiv \vec{g} \frac{m_{g}}{\left(1-V^{2} / c^{2}\right)^{3 / 2}}
\end{aligned}
$$

whence results

$$
\vec{a} \equiv \vec{g}
$$

Consequently, the equivalence is evident, and therefore the Einstein's equations from the General Relativity continue obviously valid.

The new expression for $F_{i}$ (Eqs.(5) and (6)) shows that the inertial forces are proportional to the gravitational mass, $m_{g}$. This means that these forces result from the gravitational interaction between the particle and the other gravitational masses of the Universe, just as Mach's principle predicts. Therefore the
new expression for the inertial forces incorporates the Mach's principle into Gravitation Theory, and furthermore reveals that the inertial effects upon a particle can be reduced because, as we have seen, the gravitational mass may be reduced. When $m_{g}=m_{i}$ the nonrelativistic equation for inertial forces, $\vec{F}_{i}=m_{g} \vec{a}$, reduces to $\vec{F}_{i}=m_{i} \vec{a}$. This is the well-known Newton's second law for motion.

In Einstein's Special Relativity Theory the motion of a free-particle is described by means of $\delta S=0$ [9]. Now based on Eq.(1), $\delta S=0$ will be given by the following expression

$$
\delta S=-m_{g} c \delta \int d s=0
$$

which also describes the motion of the particle inside the gravitational field. Thus, the Einstein's equations from the General Relativity can be derived starting from $\delta\left(S_{m}+S_{g}\right)=0$, where $S_{g}$ and $S_{m}$ refer to action of the gravitational field and the action of the matter, respectively [10].

The variations $\delta S_{g}$ and $\delta S_{m}$ can be written as follows[11]:
$\delta S_{g}=\frac{c^{3}}{16 \pi G} \int\left(R_{i k}-\frac{1}{2} g_{i k} R\right) \delta g^{i k} \sqrt{-g} d \Omega$
$\delta S_{m}=-\frac{1}{2 c} \int T_{i k} \delta \delta^{i k} \sqrt{-g} d \Omega$
where $R_{i k}$ is the Ricci's tensor; $g_{i k}$ the metric tensor and $T_{i k}$ the matter's energy-momentum tensor:

$$
\begin{equation*}
T_{i k}=\left(P+\varepsilon_{g}\right) \mu_{i} \mu_{k}+P g_{i k} \tag{62}
\end{equation*}
$$

where $P$ is the pressure and $\varepsilon_{g}=\rho_{g} c^{2}$ is now, the density of gravitational energy, $E_{g}$, of the particle; $\rho_{g}$ is then the density of gravitational mass of the particle, i.e., $m_{g}$ at the volume unit.

Substitution of (60) and (61) into $\delta S_{m}+\delta S_{g}=0$ yields
$\frac{c^{3}}{16 \pi G} \int\left(R_{i k}-\frac{1}{2} g_{i k} R-\frac{8 \pi G}{c^{4}} T_{i k}\right) \delta g^{i k} \sqrt{-g} d \Omega=0$ whence,

$$
\begin{equation*}
\left(R_{i k}-\frac{1}{2} g_{i k} R-\frac{8 \pi G}{c^{4}} T_{i k}\right)=0 \tag{63}
\end{equation*}
$$

because the $\delta g_{i k}$ are arbitrary.
The Eqs.(63) in the following forms;

$$
\begin{equation*}
R_{i k}-\frac{1}{2} g_{i k} R=\frac{8 \pi G}{c^{4}} T_{i k} \tag{64}
\end{equation*}
$$

or

$$
\begin{equation*}
R_{i}^{k}-\frac{1}{2} g \delta_{i}^{k} R=\frac{8 \pi G}{c^{4}} T_{i}^{k} . \tag{65}
\end{equation*}
$$

are the Einstein's equations from the General Relativity.

Making on the obtained equations for the gravitational field, the transition to the Classical Mechanics, we obtain:

$$
\begin{equation*}
\Delta \Phi=4 \pi G\left(\frac{\varepsilon_{g}}{c^{2}}\right)=4 \pi G \rho_{g} \tag{66}
\end{equation*}
$$

This is the nonrelativistic equation for the gravitational field, whose general solution is

$$
\begin{equation*}
\Phi=-G \int \frac{\varepsilon_{g} d V}{r c^{2}} \tag{67}
\end{equation*}
$$

This equation express the nonrelativistic potential of the gravitational field for any distribution of mass. In particular, for only one particle with gravitational energy $E_{g}=m_{g} c^{2}$, the result is

$$
\begin{equation*}
\Phi=-G E_{g} / r c^{2} \tag{68}
\end{equation*}
$$

Thus, the gravity $\vec{g}$ into the gravitational field created by the particle is

$$
\begin{equation*}
\vec{g}=-\frac{\partial \Phi}{\partial r}=-G \frac{E_{g}}{\vec{r}^{2} c^{2}}=-G \frac{m_{g}}{\vec{r}^{2}} . \tag{69}
\end{equation*}
$$

Therefore, the gravitational force $\vec{F}_{g}$ which acts on that field, upon another particle of gravitational mass $m_{g}^{\prime}$ is then given by:

$$
\begin{equation*}
\vec{F}_{g}=m_{g}^{\prime} \vec{g}=-G \frac{m_{g} m_{g}^{\prime}}{\vec{r}^{2}} \tag{70}
\end{equation*}
$$

If $m_{g}>0$ and $m_{g}^{\prime}<0$, or $m_{g}<0$ and $m_{g}^{\prime}>0$ the force will be repulsive; the force will never be null due to the existence of a minimum value for $m_{g}$ (see Eq. (24)). However, if $m_{g}<0$ and $m_{g}^{\prime}<0$, or $m_{g}>0$ and $m_{g}^{\prime}>0$ the force will be attractive. Just for
$m_{g}=m_{i}$ and $m_{g}^{\prime}=m_{i}^{\prime}$ we obtain the Newton's attraction law.

Since the gravitational interaction can be repulsive, besides attractive, such as the electromagnetic interaction, then the graviton must have spin 1 and not 2 . Consequently, the gravitational forces are also gauge forces because they are yield by the exchange of so-called "virtual" quanta of spin 1, such as the electromagnetic forces and the weak and strong nuclear forces.

Let us now deduce the Entropy Differential Equation starting from the Eq.(55). Comparison of the Eqs.(55) and (41) shows that $U n_{r}=\Delta p c$. For small velocities $(V \ll c), \Delta p \ll m_{i} c$, so that $U n_{r} \ll m_{i} c^{2}$. Under these circumstances, the development of the Eq.(55) in power of $\left(U n_{r} / m_{i} c^{2}\right)$, gives

$$
\begin{equation*}
m_{g}=m_{i}-\left(\frac{U n_{r}}{m_{i} c^{2}}\right)^{2} m_{i} \tag{71}
\end{equation*}
$$

In the particular case of thermal radiation, it is usual to relate the energy of the photons to temperature, through the relationship $\langle h v\rangle \approx k T$ where $k=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ is the Boltzmann's constant. Thus, in that case, the energy absorbed by the particle will be $U=\eta\langle h v\rangle \approx \eta k T$, where $\eta$ is a particle-dependent absorption/emission coefficient.
Therefore, Eq.(71) may be rewritten in the following form:

$$
\begin{equation*}
m_{g}=m_{i}-\left[\left(\frac{n_{r} \eta k}{c^{2}}\right)^{2} \frac{T^{2}}{m_{i}^{2}}\right] m_{i} \tag{72}
\end{equation*}
$$

For electrons at $T=300 \mathrm{~K}$, we have

$$
\left(\frac{n_{r} \eta k}{c^{2}}\right)^{2} \frac{T^{2}}{m_{e}{ }^{2}} \approx 10^{-17}
$$

Comparing Eq.(72) with Eq.(18), we obtain

$$
\begin{equation*}
E_{K i}=\frac{1}{2}\left(\frac{n_{r} \eta k}{c}\right)^{2} \frac{T^{2}}{m_{i}} . \tag{73}
\end{equation*}
$$

The derivative of $E_{K i}$ with respect to temperature $T$ is

$$
\begin{equation*}
\frac{\partial E_{K i}}{\partial T}=\left(n_{r} \eta k / c\right)^{2}\left(T / m_{i}\right) \tag{74}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
T \frac{\partial E_{K i}}{\partial T}=\frac{\left(n_{r} \eta k T\right)^{2}}{m_{i} c^{2}} \tag{75}
\end{equation*}
$$

Substitution of $E_{K i}=E_{i}-E_{i 0}$ into Eq.(75) gives

$$
\begin{equation*}
T\left(\frac{\partial E_{i}}{\partial T}+\frac{\partial E_{i 0}}{\partial T}\right)=\frac{\left(n_{r} \eta k T\right)^{2}}{m_{i} c^{2}} \tag{76}
\end{equation*}
$$

By comparing the Eqs.(76) and (73) and considering that $\partial E_{i 0} / \partial T=0$ because $E_{i 0}$ does not depends on $T$, the Eq.(76) reduces to

$$
\begin{equation*}
T\left(\partial E_{i} / \partial T\right)=2 E_{K i} \tag{77}
\end{equation*}
$$

However, Eq.(18) shows that $2 E_{K i}=E_{i}-E_{g}$ therefore Eq.(77) becomes

$$
\begin{equation*}
E_{g}=E_{i}-T\left(\partial E_{i} / \partial T\right) \tag{78}
\end{equation*}
$$

Here, we can identify the energy $E_{i}$ with the free-energy of the system-F and $E_{g}$ with the internal energy of the system-U, thus we can write the Eq.(78) in the following form:

$$
\begin{equation*}
U=F-T(\partial F / \partial T) \tag{79}
\end{equation*}
$$

This is the well-known equation of Thermodynamics. On the other hand, remembering $\partial Q=\partial \tau+\partial U\left(1^{\text {st }}\right.$ principle of Thermodynamics) and

$$
\begin{equation*}
F=U-T S \tag{80}
\end{equation*}
$$

(Helmholtz's function), we can easily obtain from (79), the following equation:

$$
\begin{equation*}
\partial Q=\partial \tau+T \partial S \tag{81}
\end{equation*}
$$

For isolated systems, $\partial \tau=0$, we thus have

$$
\begin{equation*}
\partial Q=T \partial S \tag{82}
\end{equation*}
$$

which is the well-know Entropy Differential Equation.

Let us now consider the Eq.(55) in the ultra-relativistic case where the inertial energy of the particle $E_{i}=M_{i} c^{2}$ is very greater than its inertial energy at rest $m_{i} c^{2}$. Comparison between
(4) and (10) leads to $\Delta p=E_{i} V / c^{2}$
which, in the ultra-relativistic case, gives $\quad \Delta p=E_{i} V / c^{2} \cong E_{i} / c \cong M_{i} c$. On the other hand, comparison between (55) and (41) shows that $U n_{r}=\Delta p c$. Thus $U n_{i}=\Delta p c \cong M_{i} c^{2} \gg m_{i} c^{2}$. Consequently, Eq.(55) reduces to

$$
\begin{equation*}
m_{g}=m_{i}-2 U n_{r} / c^{2} \tag{83}
\end{equation*}
$$

Therefore, the action for such particle, in agreement with the Eq.(2), is

$$
\begin{aligned}
S & =-\int_{t_{1}}^{t_{2}} m_{g} c^{2} \sqrt{1-V^{2} / c^{2}} d t= \\
& =\int_{t_{1}}^{t_{2}}\left(-m_{i}+2 U n_{r} / c^{2}\right) c^{2} \sqrt{1-V^{2} / c^{2}} d t= \\
& =\int_{t_{1}}^{t_{2}}\left[-m_{i} c^{2} \sqrt{1-V^{2} / c^{2}}+2 U n_{r} \sqrt{1-V^{2} / c^{2}}\right] d t .
\end{aligned}
$$

The integrant function is the Lagrangean, i.e.,
$L=-m_{i} c^{2} \sqrt{1-V^{2} / c^{2}}+2 U n_{r} \sqrt{1-V^{2} / c^{2}}$
Starting from the Lagrangean we can find the Hamiltonian of the particle, by means of the well-known general formula:

$$
H=V(\partial L / \partial V)-L .
$$

The result is:

$$
\begin{equation*}
H=\frac{m_{i} c^{2}}{\sqrt{1-V^{2} / c^{2}}}+U n_{r}\left[\frac{\left(4 V^{2} / c^{2}-2\right)}{\sqrt{1-V^{2} / c^{2}}}\right] . \tag{86}
\end{equation*}
$$

The second term on the right hand side of the Eq.(86) results from the particle's interaction with the electromagnetic field. Note the similarity between the obtained Hamiltonian and the well-known Hamiltonian for the particle in a electromagnetic field[12]:

$$
\begin{equation*}
H=m_{i} c^{2} / \sqrt{1-V^{2} / c^{2}}+Q \varphi \tag{87}
\end{equation*}
$$

in which $Q$ is the electric charge and $\varphi$, the field's scalar potential. The quantity $Q \varphi$ expresses, as we know, the particle's interaction with the electromagnetic field. Such as the second term on the right hand side of the Eq.(86).

It is therefore evident that it is the same quantity, expresses by means of different variables.

Thus, we can conclude that, in ultra-high energy conditions $\left(U n_{r} \cong M_{i} c^{2}>m_{i} c^{2}\right)$, the gravitational and electromagnetic fields can be described by the same Hamiltonian, i.e., in these circumstances they are unified!

It is known that starting from that Hamiltonian we may obtain a complete description of the electromagnetic field. This means that from the present theory for gravity we can also derive the equations of the electromagnetic field.

Due to $U n_{r}=\Delta p c \cong M_{i} c^{2}$ the second term on the right hand side of the Eq.(86) can be written as follows

$$
\begin{aligned}
& \Delta p c\left[\frac{\left(4 V^{2} / c^{2}-2\right)}{\sqrt{1-V^{2} / c^{2}}}\right]= \\
& \quad=\left[\frac{\left(4 V^{2} / c^{2}-2\right)}{\sqrt{1-V^{2} / c^{2}}}\right] M_{i} c^{2}= \\
& =Q \varphi=\frac{Q Q^{\prime}}{4 \pi \varepsilon_{0} R}=\frac{Q Q^{\prime}}{4 \pi \varepsilon_{0} r \sqrt{1-V^{2} / c^{2}}}
\end{aligned}
$$

whence

$$
\left(4 V^{2} / c^{2}-2\right) M_{i} c^{2}=\frac{Q Q^{\prime}}{4 \pi \varepsilon_{0} r}
$$

The factor $\left(4 V^{2} / c^{2}-2\right)$ becomes equal to 2 in the ultra-relativistic case, then follows that

$$
\begin{equation*}
2 M_{i} c^{2}=\frac{Q Q^{\prime}}{4 \pi \varepsilon_{0} r} \tag{88}
\end{equation*}
$$

From (44) we know that there is a minimum value for $M_{i}$ given by $M_{i(\text { min })}=m_{i(\text { min })}$. The Eq.(43) shows that $m_{g(\text { min })}=m_{i(\text { min })} \quad$ and $\quad$ Eq.(23) gives $m_{g(\text { min })}= \pm h / c L_{\max } \sqrt{8}= \pm h \sqrt{3 / 8} / c d_{\max }$. Thus we can write

$$
\begin{equation*}
M_{i(\min )}=m_{i(\min )}= \pm h \sqrt{3 / 8} / c d_{\max } \tag{89}
\end{equation*}
$$

According to (88) the value $2 M_{i(\text { min })} c^{2}$ is correlated to $\left(Q Q / 4 \pi \varepsilon_{0} r\right)_{\text {min }}=Q_{\text {min }}^{2} / 4 \pi \varepsilon_{0} r_{\text {max }}$, i.e.,

$$
\begin{equation*}
\frac{Q_{\min }^{2}}{4 \pi \varepsilon_{0} r_{\max }}=2 M_{i(\min )} c^{2} \tag{90}
\end{equation*}
$$

where $Q_{\text {min }}$ is the minimum electric charge in the Universe ( therefore equal to minimum electric charge of the quarks,
i.e., $\frac{1}{3} e$ ); $r_{\text {max }}$ is the maximum distance between $Q$ and $Q^{\prime}$, which should be equal to the so-called "diameter", $d_{c}$, of the visible Universe $\quad\left(d_{c}=2 l_{c}\right.$ where $l_{c}$ is obtained from the Hubble's law for $V=c$ , i.e., $l_{c}=c \tilde{H}^{-1}$ ). Thus from (90) we readily obtain

$$
\begin{align*}
Q_{\text {min }} & =\sqrt{\pi \varepsilon_{0} h c \sqrt{24}\left(d_{c} / d_{\max }\right)}= \\
& =\sqrt{\left(\pi \varepsilon_{0} h c^{2} \sqrt{96} \tilde{H}^{-1} / d_{\max }\right)}= \\
& =\frac{1}{3} e \tag{91}
\end{align*}
$$

whence we find

$$
d_{\max }=3.4 \times 10^{30} \mathrm{~m}
$$

This will be the maximum "diameter" that the Universe will reach. Consequently, Eq.(89) tells us that the elementary quantum of matter is

$$
m_{i(\text { min })}= \pm h \sqrt{3 / 8} / c d_{\text {max }}= \pm 3.9 \times 10^{-73} \mathrm{~kg}
$$

Now by combination of gravity and the uncertainty principle we will derive the expression of the Casimir force.

An uncertainty $\Delta m_{i}$ in $m_{i}$ produces an uncertainty $\Delta p$ in $p$ and therefore an uncertainty $\Delta m_{g}$ in $m_{g}$, which according to Eq.(41) , is given by

$$
\begin{equation*}
\Delta m_{g}=\Delta m_{i}-2\left[\sqrt{1+\left(\frac{\Delta p}{\Delta m_{i} c}\right)^{2}}-1\right] \Delta m_{i} \tag{92}
\end{equation*}
$$

From the uncertainty principle for position and momentum, we know that the product of the uncertainties of the simultaneously measurable values of corresponding position and momentum components is at least of the order of magnitude of $\hbar$, i.e.,

$$
\Delta p \Delta r \sim \hbar
$$

Substitution of $\Delta p \sim \hbar / \Delta r$ into (92) yields
$\Delta m_{g}=\Delta m_{i}-2\left[\sqrt{1+\left(\frac{\hbar / \Delta m_{i} c}{\Delta r}\right)^{2}}-1\right] \Delta m_{i}$
Therefore if

$$
\begin{equation*}
\Delta r \ll \frac{\hbar}{\Delta m_{i} c} \tag{94}
\end{equation*}
$$

then the expression (93) reduces to:

$$
\begin{equation*}
\Delta m_{g} \cong-\frac{2 \hbar}{\Delta r c} \tag{95}
\end{equation*}
$$

Note that $\Delta m_{g}$ does not depend on $m_{g}$.
Consequently, the uncertainty $\Delta F$ in the gravitational force $F=-G m_{g} m_{g}^{\prime} / r^{2}$, will be given by

$$
\begin{align*}
\Delta F & =-G \frac{\Delta m_{g} \Delta m_{g}^{\prime}}{(\Delta r)^{2}}= \\
& =-\left[\frac{2}{\pi(\Delta r)^{2}}\right] \frac{h c}{(\Delta r)^{2}}\left(\frac{G \hbar}{c^{3}}\right) \tag{96}
\end{align*}
$$

The amount $\left(G \hbar / c^{3}\right)^{1 / 2}=1.61 \times 10^{-35} \mathrm{~m}$ is called the Planck length, $l_{\text {planck }}$, (the length scale on which quantum fluctuations of the metric of the space time are expected to be of order unity). Thus, we can write the expression of $\Delta F$ as follows

$$
\begin{align*}
\Delta F & =-\left(\frac{2}{\pi}\right) \frac{h c}{(\Delta r)^{4}} l_{\text {planck }}^{2}= \\
& \left.=-\left(\frac{\pi}{480}\right) \frac{h c}{(\Delta r)^{4}}\left[\left(\frac{960}{\pi^{2}}\right)\right)_{\text {planck }}^{2}\right]= \\
& =-\left(\frac{\pi A_{0}}{480}\right) \frac{h c}{(\Delta r)^{4}} \tag{97}
\end{align*}
$$

or

$$
\begin{equation*}
F_{0}=-\left(\frac{\pi A_{0}}{480}\right) \frac{h c}{r^{4}} \tag{98}
\end{equation*}
$$

which is the expression of the Casimir force for $\left.A=A_{0}=\left(960 / \pi^{2}\right)\right)_{\text {planck }}^{2}$.

This suggests that $A_{0}$ is an elementary area related to existence of a minimum length $d_{\text {min }}=\tilde{k} l_{\text {planck }}$. What is in accordance with the quantization of space (29) which point out the existence of $d_{\text {min }}$.

One can be easily shown that the minimum area related to $d_{\text {min }}$ is the area of an equilateral triangle of side length $d_{\text {min }}$,i.e.,

$$
A_{\text {min }}=\left(\frac{\sqrt{3}}{4}\right) d_{\text {min }}^{2}=\left(\frac{\sqrt{3}}{4}\right) \tilde{k}^{2} l_{\text {planck }}^{2}
$$

On the other hand, the maximum area related to $d_{\text {min }}$ is the area of an sphere of radius $d_{\text {min }}$,i.e.,

$$
A_{\text {max }}=\pi d_{\text {min }}^{2}=\pi \tilde{k}^{2} l_{\text {planck }}^{2}
$$

Thus, the elementary area

$$
\begin{equation*}
A_{0}=\delta_{A} d_{\text {min }}^{2}=\delta_{A} \tilde{k}^{2} l_{\text {planck }}^{2} \tag{99}
\end{equation*}
$$

must have a value between $A_{\text {min }}$ and $A_{\max }$, i.e.,

$$
\frac{\sqrt{3}}{4}<\delta_{A}<\pi
$$

The previous assumption that $A_{0}=\left(960 / \pi^{2}\right) l_{\text {planck }}^{2}$ shows that $\delta_{A} \tilde{k}^{2}=960 / \pi^{2}$ what means that

$$
5.6<\tilde{k}<14.9
$$

Therefore we conclude that

$$
\begin{equation*}
d_{\min }=\tilde{k} l_{\text {planck }} \approx 10^{-34} \mathrm{~m} \tag{100}
\end{equation*}
$$

The $n$-esimal area after $A_{0}$ is

$$
\begin{equation*}
A=\delta_{A}\left(n d_{\text {min }}\right)^{2}=n^{2} A_{0} \tag{101}
\end{equation*}
$$

One can also be easily shown that the minimum volume related to $d_{\text {min }}$ is the volume of an regular tetrahedron of edge length $d_{\text {min }}$, i.e.,

$$
\Omega_{\text {min }}=\left(\frac{\sqrt{2}}{12}\right) d_{\text {min }}^{3}=\left(\frac{\sqrt{2}}{12}\right) \tilde{k}^{3} l_{\text {planck }}^{3}
$$

The maximum volume is the volume of a sphere of radius $d_{\text {min }}$, i.e.,

$$
\Omega_{\max }=\left(\frac{4 \pi}{3}\right) d_{\text {min }}^{3}=\left(\frac{4 \pi}{3}\right) \tilde{k}^{3} l_{\text {planck }}^{3}
$$

Thus, the elementary volume $\Omega_{0}=\delta_{V} d_{\text {min }}^{3}=\delta_{V} \tilde{k}^{3} l_{\text {planck }}^{3}$ must have a value between $\Omega_{\text {min }}$ and $\Omega_{\text {max }}$, i.e.,

$$
\left(\frac{\sqrt{2}}{12}\right)<\delta_{V}<\frac{4 \pi}{3}
$$

On the other hand, the $n$-esimal volume after $\Omega_{0}$ is

$$
\Omega=\delta_{V}\left(n d_{\min }\right)^{3}=n^{3} \Omega_{0} \quad n=1,2,3, \ldots, n_{\max } .
$$

The existence of $n_{\max }$ given by (26), i.e.,

$$
\begin{aligned}
n_{\max } & =L_{\max } / L_{\min }=d_{\max } / d_{\min }= \\
& =\left(3.4 \times 10^{30}\right) / \tilde{k} l_{\text {planck }} \approx 10^{64}
\end{aligned}
$$

shows that the Universe must have a finite volume whose value at the present stage is

$$
\Omega_{U p}=n_{U p}^{3} \Omega_{0}=\left(d_{p} / d_{\text {min }}\right)^{3} \delta_{V} d_{\text {min }}^{3}=\delta_{V} d_{p}^{3}
$$

where $d_{p}$ is the present length scale of the Universe. In addition as $\left(\frac{\sqrt{2}}{12}\right)<\delta_{V}<\frac{4 \pi}{3}$ we conclude that the

Universe must have a polyhedral space topology with volume between the volume of a regular tetrahedron of edge length $d_{p}$ and the volume of the sphere of diameter $d_{p}$.

A recent analysis of astronomical data suggests not only that the Universe is finite, but also that it has a dodecahedral space topology [13,14], what is in strong accordance with the theoretical predictions above.

From (22) and (26) we have that $L_{\text {max }}=d_{\text {max }} / \sqrt{3}=n_{\text {max }} d_{\text {min }} / \sqrt{3}$. Since (100) gives $d_{\text {min }} \cong 10^{-34} m$ and $n_{\text {max }} \cong 10^{64}$ we conclude that $L_{\text {max }} \cong 10^{30} \mathrm{~m}$. From the Hubble's law and (22) we have that $V_{\text {max }}=\tilde{H} l_{\text {max }}=\tilde{H}\left(d_{\text {max }} / 2\right)=(\sqrt{3} / 2) \tilde{H} L_{\text {max }}$ where $\widetilde{H}=1.7 \times 10^{-18} s^{-1}$. Therefore we obtain $V_{\max } \cong 10^{12} \mathrm{~m} / \mathrm{s}$.

Now multiplying (98) by $n^{2}$ the expression of $F_{0}$ we obtain

$$
F=n^{2} F_{0}=-\left(\frac{\pi n^{2} A_{0}}{480}\right) \frac{h c}{r^{4}}=-\left(\frac{\pi A}{480}\right) \frac{h c}{r^{4}} \text { (102) }
$$ which is the general expression of the Casimir force.

Thus we conclude that the Casimir effect is just a gravitational effect related to the uncertainty principle.

Note that the Eq.(102) arises only when $\Delta m_{i}$ and $\Delta m_{i}^{\prime}$ satisfy Eq.(94). If only $\Delta m_{i}$ satisfies Eq.(94), i.e., $\Delta m_{i} \ll \hbar / \Delta r c$ but $\Delta m_{i}^{\prime} \gg \hbar / \Delta r c$ then $\Delta m_{g}$ and $\Delta m_{g}^{\prime}$ will be respectively given by

$$
\Delta m_{g} \cong-2 \hbar / \Delta r c \quad \text { and } \quad \Delta m_{g}^{\prime} \cong \Delta m_{i}
$$

Consequently, the expression (96) becomes

$$
\begin{align*}
\Delta F & =\frac{h c}{(\Delta r)^{3}}\left(\frac{G \Delta m_{i}^{\prime}}{\pi c^{2}}\right)=\frac{h c}{(\Delta r)^{3}}\left(\frac{G \Delta m_{i}^{\prime} c^{2}}{\pi c^{4}}\right)= \\
& =\frac{h c}{(\Delta r)^{3}}\left(\frac{G \Delta E^{\prime}}{\pi c^{4}}\right) \tag{103}
\end{align*}
$$

However, from the uncertainty principle for energy and time we know that

$$
\begin{equation*}
\Delta E \sim \hbar / \Delta t \tag{104}
\end{equation*}
$$

Therefore we can write the expression (103) in the following form:

$$
\begin{align*}
\Delta F & =\frac{h c}{(\Delta r)^{3}}\left(\frac{G \hbar}{c^{3}}\right)\left(\frac{1}{\pi \Delta t^{\prime} c}\right)= \\
& =\frac{h c}{(\Delta r)^{3}} l_{\text {planck }}^{2}\left(\frac{1}{\pi \Delta t^{\prime} c}\right) \tag{105}
\end{align*}
$$

From the General Relativity Theory we know that $d r=c d t / \sqrt{-g_{00}}$. If the field is weak then $g_{00}=-1-2 \phi / c^{2}$ and $d r=c d t /\left(1+\phi / c^{2}\right)=c d t /\left(1-G m / r^{2} c^{2}\right)$.
For $G m / r^{2} c^{2} \ll 1$ we obtain $d r \cong c d t$. Thus, if $d r=d r^{\prime}$ then $d t=d t^{\prime}$. This means that we may change ( $\Delta t^{\prime} c$ ) by $(\Delta r)$ into (105). The result is

$$
\begin{aligned}
\Delta F & =\frac{h c}{(\Delta r)^{4}}\left(\frac{1}{\pi} l_{\text {planck }}^{2}\right)= \\
& =\left(\frac{\pi}{480}\right) \frac{h c}{(\Delta r)^{4}} \underbrace{-}_{\frac{\frac{1}{2} A_{0}}{\frac{480}{\pi^{2}}} l_{\text {planck }}^{2}})= \\
& =\left(\frac{\pi A_{0}}{960}\right) \frac{h c}{(\Delta r)^{4}}
\end{aligned}
$$

or

$$
F_{0}=\left(\frac{\pi A_{0}}{960}\right) \frac{h c}{r^{4}}
$$

whence

$$
\begin{equation*}
F=\left(\frac{\pi A}{960}\right) \frac{h c}{r^{4}} \tag{106}
\end{equation*}
$$

Now the Casimir force is repulsive, and its intensity is the half of the intensity previously obtained (102).

Consider the case when both $\Delta m_{i}$ and $\Delta m_{i}^{\prime}$ do not satisfy Eq.(94), and

$$
\begin{aligned}
& \Delta m_{i} \gg \hbar / \Delta r c \\
& \Delta m_{i}^{\prime} \gg \hbar / \Delta r c
\end{aligned}
$$

In this case, $\Delta m_{g} \cong \Delta m_{i}$ and $\Delta m_{g}^{\prime} \cong \Delta m_{i}^{\prime}$. Thus,

$$
\begin{aligned}
& \Delta F=-G \frac{\Delta m_{i} \Delta m_{i}^{\prime}}{(\Delta r)^{2}}=-G \frac{\left(\Delta E / c^{2}\right)\left(\Delta E^{\prime} / c^{2}\right)}{(\Delta r)^{2}}= \\
& =-\left(\frac{G}{c^{4}}\right) \frac{(\hbar / \Delta t)^{2}}{(\Delta r)^{2}}=-\left(\frac{G \hbar}{c^{3}}\right) \frac{h c}{(\Delta r)^{2}}\left(\frac{1}{c^{2} \Delta t^{2}}\right)= \\
& =-\left(\frac{1}{2 \pi}\right) \frac{h c}{(\Delta r)^{4}} l_{\text {planck }}^{2}= \\
& =-\left(\frac{\pi}{1920}\right) \frac{h c}{(\Delta r)^{4}}\left(\frac{960}{\pi^{2}} l_{\text {planck }}^{2}\right)=-\left(\frac{\pi A_{0}}{1920}\right) \frac{h c}{(\Delta r)^{4}}
\end{aligned}
$$

whence

$$
\begin{equation*}
F=-\left(\frac{\pi A}{1920}\right) \frac{h c}{r^{4}} \tag{107}
\end{equation*}
$$

The force will be attractive and its intensity will be the fourth part of the intensity given by the first expression (102) for the Casimir force.

There is a crucial cosmological problem to be solved: the problem of the hidden mass. Most theories predict that the amount of known matter, detectable and available in the universe, is only about $1 / 10$ to $1 / 100$ of the amount needed to close the universe. That is, to achieve the density sufficient to close-up the universe by maintaining the gravitational curvature (escape velocity equal to the speed of light) at the outer boundary.

The Eq.(45) may solve this problem. We will start by substituting the well-known expression of Hubble's law for velocity, $V=\tilde{H} l$, into Eq.(45). ( $\tilde{H}=1.7 \times 10^{-18} s^{-1}$ is the Hubble constant). The expression obtained shows that particles which are at distances $\quad l=l_{0}=(\sqrt{5} / 3)(c / \tilde{H})=1.3 \times 10^{26} m$ have quasi null gravitational mass $m_{g}=m_{g(\text { min })}$; beyond this distance, the particles have negative gravitational mass. Therefore, there are two welldefined regions in the Universe; the region of the bodies with positive gravitational masses and the region of the bodies with negative gravitational mass. The total gravitational mass of the first
region, in accordance with Eq.(45), will be given by

$$
M_{g 1} \cong M_{i 1}=\frac{m_{i 1}}{\sqrt{1-\bar{V}_{1}^{2} / c^{2}}} \cong m_{i 1}
$$

where $m_{i 1}$ is the total inertial mass of the bodies of the mentioned region; $\bar{V}_{1} \ll c$ is the average velocity of the bodies at region 1. The total gravitational mass of the second region is

$$
M_{g 2}=M_{i 2}-2\left(\frac{1}{\sqrt{1-{\overline{V_{2}^{2}}}^{2} / c^{2}}}-1\right) M_{i 2}
$$

where $\quad \bar{V}_{2}$ is the average velocity of the bodies ; $M_{i 2}=m_{i 2} / \sqrt{1-\bar{V}_{2}^{2} / c^{2}}$ and $m_{i 2}$ is the total inertial mass of the bodies of region 2.

Now consider that from Eq.(7), we can write

$$
\xi=\frac{E_{g}}{V}=\frac{M_{g} c^{2}}{V}=\rho_{g} c^{2}
$$

where $\xi$ is the energy density of matter.
Note that the expression of $\xi$ only reduces to the well-known expression $\rho c^{2}$, where $\rho$ is the sum of the inertial masses per volume unit, when $m_{g}=m_{i}$. Therefore, in the derivation of the well-known difference

$$
\begin{equation*}
\frac{8 \pi G \rho_{U}}{3}-\tilde{H}^{2} \tag{108}
\end{equation*}
$$

which gives the sign of the curvature of the Universe [15], we must use $\xi=\rho_{g U} c^{2}$ instead of $\xi=\rho_{U} c^{2}$. The result obviously is

$$
\begin{equation*}
\frac{8 \pi G \rho_{g U}}{3}-\tilde{H}^{2} \tag{109}
\end{equation*}
$$

where

$$
\begin{equation*}
\rho_{g U}=\frac{M_{g U}}{V_{U}}=\frac{M_{g 1}+M_{g 2}}{V_{U}} \tag{110}
\end{equation*}
$$

$M_{g U}$ and $V_{U}$ are respectively the total gravitational mass and the volume of the Universe.
Substitution of $M_{g 1}$ and $M_{g 2}$ into expression (110) gives
$\rho_{g U}=\frac{m_{i U}+\left[\left(\frac{3}{\sqrt{1-\bar{V}_{2}^{2} / c^{2}}}-\frac{2}{1-\bar{V}_{2}^{2} / c^{2}}\right) m_{i 2}-m_{i 2}\right]}{V_{U}}$
where $m_{i U}=m_{i 1}+m_{i 2}$ is the total inertial mass of the Universe.

The volume $V_{1}$ of the region 1 and the volume $V_{2}$ of the region 2, are respectively given by
$V_{1}=2 \pi^{2} l_{0}^{3} \quad$ and $\quad V_{2}=2 \pi^{2} l_{c}^{3}-V_{1}$ where $\quad l_{c}=c / \tilde{H}=1.8 \times 10^{26} \mathrm{~m}$ is the so-called "radius" of the visible Universe. Moreover, $\rho_{i 1}=m_{i 1} / V_{1}$ and $\rho_{i 2}=m_{i 2} / V_{2}$. Due to the hypothesis of the uniform distribution of matter in the space, follows that $\rho_{i 1}=\rho_{i 2}$. Thus, we can write

$$
\frac{m_{i 1}}{m_{i 2}}=\frac{V_{1}}{V_{2}}=\left(\frac{l_{0}}{l_{c}}\right)^{3}=0.38
$$

Similarly,

$$
\frac{m_{i U}}{V_{U}}=\frac{m_{i 2}}{V_{2}}=\frac{m_{i 1}}{V_{1}}
$$

Therefore,
$m_{i 2}=\frac{V_{2}}{V_{U}} m_{i U}=\left[1-\left(\frac{l_{0}}{l_{c}}\right)^{3}\right] m_{i U}=0.62 m_{i U}$
and $m_{i 1}=0.38 m_{i U}$.
Substitution of $m_{i 2}$ into the expression of $\rho_{g U}$ yields
$\rho_{g U}=\frac{m_{U}+\left[\left(\frac{1.86}{\sqrt{1-\bar{V}_{2}^{2} / c^{2}}}-\frac{1.24}{1-\bar{V}_{2}^{2} / c^{2}}-0.62\right) m_{U U}\right]}{V_{U}}$
Due to $\overline{V_{2}} \cong c$, we conclude that the term between bracket (hidden mass) is very greater than $10 m_{i U}$. The amount $m_{i U}$ is the mass of known matter in the universe ( $1 / 10$ to $1 / 100$ of the amount needed to close the Universe).

Consequently, the total mass
$m_{i U}+\left[\left(\frac{1.86}{\sqrt{1-\bar{V}_{2}^{2} / c^{2}}}-\frac{1.24}{1-\bar{V}_{2}^{2} / c^{2}}-0.62\right) m_{i U}\right]$ must be sufficient to close the

Universe. This solves therefore the problem of the hidden mass.

There is another cosmological problem to be solved: the problem of the anomalies in the spectral red-shift of certain galaxies and stars.

Several observers have noticed red-shift values that cannot be explained by the Doppler-Fizeau effect or by the Einstein effect (the gravitational spectrum shift, supplied by Einstein's theory).

This is the case of the so-called Stefan's quintet (a set of five galaxies which were discovered in 1877), whose galaxies are located at approximately the same distance from the Earth, according to very reliable and precise measuring methods. But, when the velocities of the galaxies are measured by its red-shifts, the velocity of one of them is much greater than the velocity of the others.

Similar observations have been made on the Virgo constellation and spiral galaxies. Also the Sun presents a red-shift greater than the predicted value by the Einstein effect.

It seems that some of these anomalies can be explained if we consider the Eq.(45) in the calculation of the gravitational mass of the point of emission.

The expression of the gravitational spectrum shift, supplied by Einstein's theory [16] is given by

$$
\begin{align*}
\Delta \omega & =\omega_{1}-\omega_{2}=\frac{\phi_{2}-\phi_{1}}{c^{2}} \omega_{1}= \\
& =\frac{-G m_{y_{2}} / r_{2}+G m_{g_{1}} / r_{1}}{c^{2}} \omega_{1} \tag{111}
\end{align*}
$$

where $\omega_{1}$ is the frequency of the light at the point of emission ; $\omega_{2}$ is the frequency at the point of observation; $\phi_{1}$ and $\phi_{2}$ are respectively, the Newtonian gravitational potentials at the point of emission and at the point of observation.

This expression has been deduced from $t=t_{0} \sqrt{-g_{00}}$ [17] which
correlates own time (real time), $t$, with the temporal coordinate $\mathrm{x}^{0}$ of the space-time ( $t_{0}=x^{0} / c$ ).

When the gravitational field is weak, the temporal component $g_{00}$ of the metric tensor is given by $g_{o o}=-1-2 \phi / c^{2}[18]$. Thus we readily obtain

$$
\begin{equation*}
t=t_{0} \sqrt{1-2 G m_{g} / r c^{2}} \tag{112}
\end{equation*}
$$

Curiously, this equation tell us that we can have $t<t_{0}$ when $m_{g}>0$; and $t>t_{0}$ for $m_{g}<0$. In addition, if $m_{g}=c^{2} r / 2 G$, i.e., if $r=2 G m_{g} / c^{2}$ (Schwarzschild radius) we obtain $t=0$.

Let us now consider the wellknown process of stars' gravitational contraction. It is known that the destination of the star is directly correlated to its mass. If the star's mass is less than $1.4 \mathrm{M}_{\odot}$ ( Schemberg-Chandrasekhar's limit), it becomes a white dwarf. If its mass exceeds that limit, the pressure produced by the degenerate state of the matter no longer counterbalances the gravitational pressure, and the star's contraction continues. Afterwards occur the reactions between protons and electrons ( capture of electrons ), where neutrons and anti-neutrinos are produced.

The contraction continues until the system regains stability ( when the pressure produced by the neutrons is sufficient to stop the gravitational collapse). Such systems are called neutron stars.

There is also a critical mass for the stable configuration of neutron stars. This limit has not been fully defined as yet, but it is known that it is located between $1.8 \mathrm{M}_{\odot}$ and $2.4 \mathrm{M}_{\odot}$. Thus, if the mass of the star exceeds $2.4 \mathrm{M}_{\odot}$, the contraction will continue.

According to Hawking[19] collapsed objects cannot have mass
less than $\sqrt{\hbar c / 4 G}=1.1 \times 10^{-8} \mathrm{~kg}$. This
means that the neutrons cluster becomes a cluster of superparticles where the minimal inertial mass of the superparticle is

$$
\begin{equation*}
m_{i(s p)}=1.1 \times 10^{-8} \mathrm{~kg} . \tag{113}
\end{equation*}
$$

Symmetry is a fundamental attribute of the Universe that enables an investigator to study particular aspects of physical systems by themselves. For example, the assumption that space is homogeneous and isotropic is based on Symmetry Principle. Also here, by symmetry, we can assume that there are only superparticles with mass $m_{i(p)}=1.1 \times 10^{-8} \mathrm{~kg}$ in the cluster of superparticles.

Let us now imagine the Universe coming back for the past. There will be an instant in which it will be similar to a neutrons cluster, such as the stars at the final state of gravitational contraction. Thus, with the progressing of the compression, the neutrons cluster becomes a cluster of superparticles. Obviously, this only can occur before $10^{-23}$ (after the Big-Bang).

The temperature T of the Universe at the $10^{-43} \mathrm{~s}<\mathrm{t}<10^{-23} \mathrm{~s}$ period can be calculated by means of the well-known expression[20]:

$$
\begin{equation*}
T \approx 10^{22}\left(t / 10^{-23}\right)^{-1 / 2} \tag{114}
\end{equation*}
$$

Thus at $t \cong 10^{-43} \mathrm{~s}$ ( at the first spontaneous breaking of symmetry ) the temperature was $T \approx 10^{32} K$ $\left(\sim 10^{19} \mathrm{GeV}\right)$. Therefore, we can assume that the absorbed electromagnetic energy by each superparticle, before $t \cong 10^{-43} s$, was $U=\eta k T>1 \times 10^{9} J$ (see Eqs.(71) and (72) ). By comparing with $m_{i((p))^{2}} \cong 9 \times 10^{8} J$, we conclude that $U>m_{i(s p)}$. Therefore, the unification condition $\quad\left(U n_{r} \cong M_{i} c^{2}>m_{i} c^{2}\right) \quad$ is
satisfied. This means that, before $t \cong 10^{-43} s$, the gravitational and electromagnetic interactions were unified.

From the unification condition $\left(U n_{r} \cong M_{i} c^{2}\right)$, we may conclude that the superparticles' relativistic inertial mass $M_{i(s p)}$ is

$$
\begin{equation*}
M_{i(s p)} \cong \frac{U n_{r}}{c^{2}}=\frac{\eta n_{r} k T}{c^{2}} \approx 10^{-8} \mathrm{~kg} \tag{115}
\end{equation*}
$$

Comparing with the superparticles' inertial mass at rest (113), we conclude that

$$
\begin{equation*}
M_{i(s p)} \approx m_{i(s p)}=1.1 \times 10^{-8} \mathrm{~kg} \tag{116}
\end{equation*}
$$

From Eqs.(83) and (115), we obtain the superparticle's gravitational mass at rest, i.e.,

$$
\begin{align*}
m_{g(s p)} & =m_{i(s p)}-2 M_{i(s p)} \cong \\
& \cong-M_{i(s p)} \cong-\frac{\eta n_{r} k T}{c^{2}} \tag{117}
\end{align*}
$$

and consequently, the superparticle's relativistic gravitational mass, is

$$
\begin{equation*}
M_{g(s p)}=-\frac{\eta n_{r} k T}{c^{2} \sqrt{1-V^{2} / c^{2}}} \tag{118}
\end{equation*}
$$

Thus, the gravitational forces between two superparticles , according to (13), is given by:

$$
\begin{align*}
\vec{F}_{12} & =-\vec{F}_{21}=-G \frac{M_{g(s p)} M_{g(s p)}^{\prime}}{r^{2}} \hat{\mu}_{21}= \\
& =\left[\left(\frac{M_{i(s p)}}{m_{i(s p)}}\right)^{2}\left(\frac{G}{c^{5} \hbar}\right)\left(\eta \eta_{r} \kappa T\right)^{2}\right] \frac{\hbar c}{r^{2}} \hat{\mu}_{21} \tag{119}
\end{align*}
$$

Due to the unification of the gravitational and electromagnetic interactions at that period, we have

$$
\begin{align*}
\vec{F}_{12} & =-\vec{F}_{21}=G \frac{M_{g(s p)} M_{g(s p)}^{\prime}}{r^{2}} \hat{\mu}_{21}= \\
& =\left[\left(\frac{M_{i(s p)}}{m_{i(s p)}}\right)^{2}\left(\frac{G}{c^{5} \hbar}\right)(\eta \kappa T)^{2}\right] \frac{\hbar c}{r^{2}} \hat{\mu}_{21}= \\
& =\frac{e^{2}}{4 \pi \varepsilon_{0} r^{2}} \tag{120}
\end{align*}
$$

From the equation above we can write
$\left[\left(\frac{M_{i(s p)}}{m_{i(s p)}}\right)^{2}\left(\frac{G}{c^{5} \hbar}\right)(\eta \kappa T)^{2} \hbar c\right]=\frac{e^{2}}{4 \pi \varepsilon_{0}}$
Now assuming that
$\left(\frac{M_{i(s p)}}{m_{i(s p)}}\right)^{2}\left(\frac{G}{c^{5} \hbar}\right)(\eta \kappa T)^{2}=\psi$
the Eq.(121) can be rewritten in the following form:

$$
\begin{equation*}
\psi=\frac{e^{2}}{4 \pi \varepsilon_{0} \hbar c}=\frac{1}{137} \tag{123}
\end{equation*}
$$

which is the well-known reciprocal fine structure constant .

For $T=10^{32} K$ the Eq.(122) gives
$\psi=\left(\frac{M_{i(s p)}}{m_{i(s p)}}\right)^{2}\left(\frac{G}{c^{5} \hbar}\right)\left(\eta \eta_{r} \kappa T\right)^{2} \approx \frac{1}{100}$ (124) This value has the same order of magnitude that the exact value(1/137) of the reciprocal fine structure constant.

From equation (120) we can write:

$$
\begin{equation*}
\left(G \frac{M_{g(s p)} M_{g(s p)}^{\prime}}{\psi c \vec{r}}\right) \vec{r}=\hbar \tag{125}
\end{equation*}
$$

The term between parenthesis has the same dimensions that the linear momentum $\vec{p}$. Thus Eq.(125) tells us that

$$
\begin{equation*}
\vec{p} \cdot \vec{r}=\hbar . \tag{126}
\end{equation*}
$$

A component of the momentum of a particle cannot be precisely specified without loss of all knowledge of the corresponding component of its position at that time ,i.e., a particle cannot precisely localized in a particular direction without loss of all knowledge of its momentum component in that direction. This means that in intermediate cases the product of the uncertainties of the simultaneously measurable values of corresponding position and momentum components is at least of the order of magnitude of $\hbar$,i.e.,

$$
\begin{equation*}
\Delta p \cdot \Delta r \geq \hbar \tag{127}
\end{equation*}
$$

This relation, directly obtained here from the Unified Theory, is the well-
known relation of the Uncertainty Principle for position and momentum.

According to Eq.(83), the gravitational mass of the superparticles at the center of the cluster becomes negative when $2 \eta_{n_{r}} k T / c^{2}>m_{i(s p)}$, i.e., when

$$
T>T_{\text {critical }}=\frac{m_{i(p)}}{2 \eta n_{r} k} \approx 10^{32} \mathrm{~K} .
$$

According to Eq.(114) this temperature corresponds to $t_{c} \approx 10^{-43} \mathrm{~s}$.

With the progressing of the compression, more superparticles into the center will have negative gravitational mass. Consequently, there will have a critical point in which the repulsive gravitational forces between the superparticles with negative gravitational masses and the superparticles with positive gravitational masses will be so strong that an explosion will occur. This is the experiment that we call the Big Bang.

Now, starting from the Big Bang to the present time. Immediately after the Big Bang, the superparticles' decompressing begins. The gravitational mass of the most central superparticle will only be positive when the temperature becomes smaller than the critical temperature, $T_{\text {critical }} \approx 10^{32} \mathrm{~K}$. At the maximum state of compression ( exactly at the Big Bang ) the volumes of the superparticles was equal to the elementary volume $\Omega_{0}=\delta_{V} d_{\text {min }}$ and the volume of the Universe was $\Omega=\delta_{V}\left(n d_{\text {min }}\right)^{3}=\delta_{V} d_{\text {initial }}^{3}$ where $d_{\text {initial }}$ was the initial length scale of the Universe. At this very moment the average density of the Universe was equal to the average density of the superparticles, thus we can write

$$
\begin{equation*}
\left(\frac{d_{\text {initial }}}{d_{\text {min }}}\right)^{3}=\frac{M_{i(U)}}{m_{i(s p)}} \tag{128}
\end{equation*}
$$

where $M_{i(U)} \approx 10^{53} \mathrm{~kg}$ is the inertial mass of the Universe. It has already been
shown that $d_{\text {min }}=\tilde{k} l_{\text {planck }} \approx 10^{-34} \mathrm{~m}$. Then, from the Eq.(128), we obtain:

$$
\begin{equation*}
d_{\text {initial }} \approx 10^{-14} \mathrm{~m} \tag{129}
\end{equation*}
$$

After the Big Bang the Universe expands itself from $d_{\text {initial }}$ up to $d_{c r}$ ( when the temperature decreasing reaches the critical temperature $T_{\text {critical }} \approx 10^{32} \mathrm{~K}$, and the gravity becomes attractive). Thus, it expands by $d_{c r}-d_{\text {initial }}$, under effect of the repulsive gravity

$$
\begin{aligned}
\bar{g} & =\sqrt{g_{\text {max }} g_{\text {min }}}= \\
& =\sqrt{\left[G\left(\frac{1}{2} M_{g(U)}\right) /\left(\frac{1}{2} d_{\text {initiai }}\right)^{2}\right]\left[G \frac{1}{2} M_{i(U)} /\left(\frac{1}{2} d_{c r}\right)^{2}\right]}= \\
& =\frac{2 G \sqrt{M_{g(U)} M_{i(U)}}}{d_{c c} d_{\text {initial }}}=\frac{2 G \sqrt{\sum m_{g(s p)} M_{i(U)}}}{d_{c r} d_{\text {initial }}}= \\
& =\frac{2 G \sqrt{\chi \sum m_{i(s p)} M_{i(U)}}}{d_{c r} d_{\text {initial }}}=\frac{2 G M_{i(U)} \sqrt{\chi}}{d_{c r} d_{\text {initial }}}
\end{aligned}
$$

during to a time $t_{c} \approx 10^{-43} s$.Thus,

$$
\begin{equation*}
d_{c r}-d_{\text {initial }}=\frac{1}{2} \bar{g}\left(t_{c}\right)^{2}=(\sqrt{\chi}) \frac{G M_{i(U)}}{d_{c r} d_{\text {initial }}}\left(t_{c}\right)^{2} \tag{130}
\end{equation*}
$$

The Eq.(83), gives

$$
\chi=\frac{m_{g(s p)}}{m_{i(s p)}}=1-\frac{2 U n_{r}}{m_{i(s p)} c^{2}}=1-\frac{2 \eta n_{r} k T}{m_{i(s p)} c^{2}} \approx 10^{-32} T
$$

The temperature at the beginning of the Big Bang ( $\mathrm{t}=0$ ) should have been very greater than $T_{\text {critical }} \approx 10^{32} \mathrm{~K}$. Thus, $\chi$ must be a very big number. Then it is easily seen that during this period, the Universe expanded at an astonishing rate. Thus, there is an evident inflation period, which ends at $t_{c} \approx 10^{-43} \mathrm{~s}$.

With the progressing of the decompression, the superparticles cluster becomes a neutrons cluster. This means that the neutrons are created without its antiparticle, the antineutron. Thus it solves the matter/antimatter dilemma that is unresolved in many cosmologies.

Now a question: How did the primordial superparticles appear at the beginning of the Universe?

It is a proven quantum fact that a wave function $\Psi$ may collapse and that at this moment all the possibilities that it describes are suddenly expressed in reality. This means that, through this process, particles can be suddenly materialized.

The materialization of the primordial superparticles into a critical volume denotes knowledge of what would happen starting from that initial condition, fact that points towards the existence of a Creator.

## CONCLUSION

We have described a coherent way for the quantization of gravity, which provides a consistent unification of gravity with electromagnetism. As we have seen, this new approach will allow us to understand some crucial matters in Quantum Cosmology.

The equation of correlation between gravitational and inertial masses, which has been derived directly from the theory of gravity, has relevant technological consequences. We have seen that gravitational mass can be negative at specific conditions. This means that it will be possible to build gravitational binaries (gravitational motors), and to extract energy from any site of a gravitational field. Obviously, the Gravity Control will be also very important to Transportation Systems. On the other hand, negative gravitational mass suggests the possibility of dipole gravitational radiation. This fact is highly relevant because now we may build transceivers to operate with gravitational waves. Furthermore, the receiver would allow us to directly observe for the first time the Cosmic Microwave Background in Gravitational Radiation, which would picture the Universe at the beginning of the BigBang.

## APPENDIX A

It is known that the orbital angular momentum $\vec{L}$ and the intrinsic spin angular momentum $\vec{S}$ interact magnetically between themselves to produce a total angular momentum $\vec{J}$. We then have

$$
\begin{align*}
& L=\hbar \sqrt{l(l+1)}  \tag{A1}\\
& L_{z}=m_{l} \hbar  \tag{A2}\\
& S=\hbar \sqrt{s(s+1)}  \tag{A3}\\
& S_{z}=m_{s} \hbar  \tag{A4}\\
& J=\hbar \sqrt{j(j+1)}  \tag{A5}\\
& J_{z}=m_{j} \hbar \tag{A6}
\end{align*}
$$

where $l$ is the orbital quantum number, $s$ is the spin quantum number, $j=l \pm s \quad$ is the total quantum number; $m_{l}$ is the orbital magnetic quantum number; $m_{s}$ is the spin magnetic quantum number and $m_{j}=m_{l} \pm m_{s}$ is the total magnetic number.

Also atomic nuclei have intrinsic angular momenta that contribute to the total angular momenta of the atoms. However, this contribution is negligible [21].

The magnetic moment $\vec{M}_{j}$ associated with $\vec{J}$ is [22]:

$$
\begin{equation*}
\vec{M}_{j}=\gamma(e / 2 m) \vec{J} \tag{A7}
\end{equation*}
$$

where $\gamma$ is called the gyromagnetic ratio. The values of $\gamma$ for electrons, neutrons and protons are respectively: - 2.0024; -3.8256 and 5.5851. For nuclear protons,
$l=m_{l}=1, \quad s=m_{s}=1 / 2, \quad j=l-s=1 / 2, \quad m_{j}=$ $m_{l}-m_{s}=1 / 2, \quad J_{p}=(\sqrt{3} / 2) \hbar, \quad J_{p z}=(1 / 2) \hbar$.
Thus, Eq.(A7) gives
$M_{J p}=\gamma_{p}\left(e / 2 m_{p}\right) J_{p}=2.44 \times 10^{-26}$ A. $m^{2} \quad$ (A8)
$M_{J p z}=\gamma_{p}\left(e / 2 m_{p}\right) J_{p z}=1.41 \times 10^{-26} A \cdot m^{2} \quad(A 9)$
Then the total magnetic field $\vec{H}_{J_{p}}$ of the proton is [23]:

$$
\begin{equation*}
\vec{H}_{J_{p}}=\vec{M}_{J_{p}} / 2 \pi r^{3} \tag{A10}
\end{equation*}
$$

where $r$ is the spin radius of the proton. The total magnetic field induction is $\vec{B}_{J_{p}}=\mu_{0} \vec{H}_{J_{p}}$, therefore

$$
\begin{equation*}
B_{J p}=\mu_{0} H_{J_{p}}=4.87 \times 10^{-33} / r^{3} \tag{A11}
\end{equation*}
$$

We can write the angular momentum $\vec{J}_{p}$ in the following form


Fig. 1
$J_{p}=K_{p} / \omega_{p}$ where $\omega_{p}$ is the angular velocity of the proton and $K_{p}$ its angular kinetic energy, given by

$$
\begin{equation*}
K_{p}=\frac{I_{p} c^{2}}{r^{2}}\left(\frac{1}{\sqrt{1-V_{p}^{2} / c^{2}}}-1\right) \tag{A12}
\end{equation*}
$$

where $V_{p}=\omega_{p} r$ and

$$
\begin{align*}
& I_{p}=\beta m_{p} r_{p}^{2}+m_{p} r^{2}= \\
& \quad=\left(\beta r_{p}^{2} / r^{2}+1\right) m_{p} r^{2}=\chi m_{p} r^{2} \tag{A13}
\end{align*}
$$

is the moment of inertia with respect to the rotation axis; $\beta$ is a numerical factor, which depends on the structural form adopted for the proton. In the case of a solid sphere, $\beta=\frac{2}{5}$.

For $V \ll c$, Eq.(A12) reduces to $K=\left(I c^{2} / r^{2}\right)\left(V^{2} / 2 c^{2}\right)=1 / 2 I\left(V^{2} / r^{2}\right)=1 / 2 I \omega^{2}$
Comparison of $J_{p}=(\sqrt{3} / 2) \hbar$ and $J_{p}=K_{p} / \omega_{p}$ shows that

$$
\begin{equation*}
\omega_{p}=\frac{\chi_{p} m_{p} c^{2}}{(\sqrt{3} / 2) \hbar}\left(\frac{1}{\sqrt{1-V_{p}^{2} / c^{2}}}-1\right) \tag{A14}
\end{equation*}
$$

From the intrinsic precession motion of the proton we observe that
(Fig.1) the component of the Lorentz's force, $F=e V_{p} B_{J_{p} s} \sin \phi=e V_{p} B_{J_{p} p}$, about the $x$ axis, $F \cos \alpha$, is equal to the centripetal force $F_{c p}=m_{p} V_{p}^{2} / r=I \omega^{2} / r=\chi_{p} m_{p} V_{p} \omega_{p}$. Thus,

$$
\begin{equation*}
\left(e V_{p} B_{J_{p z}}\right) \cos \alpha=\chi_{p} m_{p} V_{p} \omega_{p} \tag{A15}
\end{equation*}
$$

However $\quad B_{J p z}=B_{J p} \cos \alpha$. Therefore the equation above becomes

$$
\begin{equation*}
B_{J p}=\frac{\chi_{p} m_{p} \omega_{p}}{e \cos ^{2} \alpha} \tag{A16}
\end{equation*}
$$

Comparison of (A16) and (A11) shows that

$$
\begin{equation*}
V_{p}=4.66 \times 10^{-25} \frac{\cos ^{2} \alpha}{\chi_{p} r^{2}} \tag{A17}
\end{equation*}
$$

Due to $V_{p}<c$, it follows from Eq.(A17) that

$$
\begin{equation*}
r>\left(\frac{6.83 \times 10^{-13}}{\sqrt{c}}\right) \frac{\cos \alpha}{\sqrt{\chi_{p}}}=\frac{0.028 \cos \alpha}{\sqrt{\chi_{p}}} r_{p} \tag{A18}
\end{equation*}
$$

where $r_{p}=1.4 \times 10^{-15} \mathrm{~m}$ and

$$
\cos \alpha=J_{p z} / J_{p}=0.577
$$

On the other hand, from the nuclear dimensions we know that $r<r_{p}=1.4 \times 10^{-15} \mathrm{~m}$. Thus we can write

$$
\begin{equation*}
\frac{0.028 \cos \alpha}{\sqrt{\chi_{p}}} r_{p}<r<r_{p} \tag{A19}
\end{equation*}
$$

An approximated value for $r$ can be obtained by means of the following relation

$$
\begin{equation*}
r=\frac{r_{p}}{Y}=Y\left(\frac{0.028 \cos \alpha}{\sqrt{\chi_{p}}} r_{p}\right) \tag{A20}
\end{equation*}
$$

whence

$$
Y=7.87 \sqrt[4]{\chi_{p}}
$$

Therefore we can write

$$
\begin{equation*}
r=\frac{r_{p}}{Y}=1.78 \times 10^{-16} / \sqrt[4]{\chi_{p}} \tag{A21}
\end{equation*}
$$

or

$$
\begin{equation*}
\chi_{p}=\frac{1.0 \times 10^{-63}}{r^{4}} \tag{A22}
\end{equation*}
$$

From (A13) we have $\chi_{p} r^{2} \cong 0.1 r_{p}^{2}$ since $r<r_{p}$. Thus from (A22) we obtain

$$
\begin{equation*}
r \cong 7 \times 10^{-17} \mathrm{~m} \tag{A23}
\end{equation*}
$$

and

$$
\begin{equation*}
\chi_{p} \cong 41 \tag{A24}
\end{equation*}
$$

Comparison of $\omega_{p}=V_{p} / r$ and (A14) shows that

$$
\begin{equation*}
\frac{V_{p}}{\left(\frac{1}{\sqrt{1-V_{p}^{2} / c^{2}}}-1\right)}=4.7 \times 10^{9} \tag{A25}
\end{equation*}
$$

whence

$$
\begin{equation*}
V_{p}=3.8 \times 10^{7} \mathrm{~m} / \mathrm{s} \tag{A26}
\end{equation*}
$$



Fig. 2
Now consider a sample of a monatomic substance ( atomic number $Z$ ) inside an external magnetic field of induction $\vec{B}$. Under these circumstances the nuclear protons carry out a precession motion with respect to the direction of $\vec{B}$, making an angle $\theta$ with $\vec{B}$ (Fig.2). The binary $\tau_{p}$ which tends to align the axis of $\vec{L}_{p}$ with the axis of $\vec{B}$ is

$$
\begin{equation*}
\tau_{p}=M_{J p z} B \sin \theta \tag{A27}
\end{equation*}
$$

where $M_{J p z}=1.41 \times 10^{-26} A . \mathrm{m}^{2}$.
Therefore the magnetic field $\vec{B}_{J_{p}}$ of each nuclear proton is aligned at the same direction of $\vec{L}_{p}$, and consequently the magnetic field of the nucleus, $\vec{B}_{N}$, due to the protons at the direction of $\vec{L}_{p}$, will be given by

$$
\begin{equation*}
\vec{B}_{N}=Z \vec{B}_{J_{p}} \tag{A28}
\end{equation*}
$$

Only the component, $B_{N} \sin \theta$, alters the radius $r$ (Fig.3). The force $F_{r}$, produced by $\vec{B}_{N} \sin \theta$ and $\vec{V}_{p}$, increases the centripetal force $F=m_{p} V_{p}^{2} / r \quad$ decreasing $\quad r \quad$ and increasing $\vec{V}_{p}$. This means that the proton acquires a new velocity $V_{p}^{\prime}=\omega_{p}^{\prime} r^{\prime} \approx c$ which corresponds to a magnetic field $\vec{B}_{J_{p}}^{\prime}$ through the proton that is equal to

$$
\begin{equation*}
B_{J_{p}}^{\prime}=B_{N} \sin \theta=Z B_{J_{p}} \sin \theta \tag{A29}
\end{equation*}
$$

This phenomenon is similar to a strong compression upon the proton, which decrease $r$ increasing $V_{p}$. Equation (45) tells us that, if $V_{p}>0.745 c$ the gravitational mass of the proton becomes negative. This explains the negative gravitational masses of the superparticles, under strong thermal compression, in the neutron clusters previously mentioned.

From Eqs.(A16) and (A14) we have

$$
\begin{align*}
B_{J_{p}} & =\frac{\chi_{p} m_{p} \omega_{p}}{e \cos ^{2} \alpha}= \\
& =\frac{\chi_{p}^{2} m_{p}^{2} c^{2}}{e(\sqrt{3} / 2) \hbar \cos ^{2} \alpha}\left(\frac{1}{\sqrt{1-V_{p}^{2} / c^{2}}}-1\right) \tag{A30}
\end{align*}
$$



Fig. 3

$$
\begin{align*}
B_{J_{p}}^{\prime} & =\frac{\chi_{p} m_{p} \omega_{p}^{\prime}}{e \cos ^{2} \alpha^{\prime}}= \\
& =\frac{\chi_{p}^{2} m_{p}^{2} c^{2}}{e(\sqrt{3} / 2) \hbar \cos ^{2} \alpha^{\prime}}\left(\frac{1}{\sqrt{1-V_{p}^{\prime 2} / c^{2}}}-1\right) \tag{A31}
\end{align*}
$$

whence

$$
\begin{equation*}
\frac{B_{J_{p}}^{\prime}}{B_{J p}}=\frac{\cos ^{2} \alpha\left(\frac{1}{\sqrt{1-V_{p}^{\prime 2} / c^{2}}}-1\right)}{\cos ^{2} \alpha\left(\frac{1}{\sqrt{1-V_{p}^{2} / c^{2}}}-1\right)} \tag{A32}
\end{equation*}
$$

Comparison of (A32) and (A29) gives
$\left(\frac{1}{\sqrt{1-\left(V_{p}^{\prime 2} / c^{2}\right)}}-1\right)=$
$=\left(\frac{1}{\sqrt{1-\left(V_{p}^{2} / c^{2}\right)}}-1\right) \frac{Z \sin \theta \cos ^{2} \alpha^{\prime}}{\cos ^{2} \alpha}$
Substitution of (A33) into (45) shows that the gravitational mass of the nuclear protons can be written as follows
$m_{g p}=m_{p}-2\left(\frac{1}{\sqrt{1-V_{p}^{2} / c^{2}}}-1\right) \frac{Z \sin \theta \cos ^{2} \alpha}{\cos ^{2} \alpha} m_{p}$ (A34) From (A14) we can write

$$
\begin{equation*}
\frac{r^{\prime}}{r}=\frac{V_{p}^{\prime} / \omega_{p}^{\prime}}{V_{p} / \omega_{p}}=\left(\frac{V_{p}^{\prime}}{V_{p}}\right)\left(\frac{\left.\frac{1}{\sqrt{1-V_{p}^{2} / c^{2}}}-1\right)}{\left(\frac{1}{\sqrt{1-V_{p}^{\prime 2} / c^{2}}}-1\right)}\right. \tag{A35}
\end{equation*}
$$

Comparison of (A35) and (A32) shows that

$$
\begin{equation*}
\frac{B_{J_{p}}^{\prime}}{B_{J_{p}}}\left(\frac{\cos ^{2} \alpha^{\prime}}{\cos ^{2} \alpha}\right)=\left(\frac{r}{r^{\prime}}\right)\left(\frac{V_{p}^{\prime}}{V_{p}}\right) \tag{A36}
\end{equation*}
$$

Substitution of (A29) into (A36) gives

$$
\begin{equation*}
\frac{r}{r^{\prime}}=Z \sin \theta\left(\frac{V_{p}}{V_{p}^{\prime}}\right) \frac{\cos ^{2} \alpha^{\prime}}{\cos ^{2} \alpha} \tag{A37}
\end{equation*}
$$

It can be easily shown that $\tan \alpha^{\prime}=\left(r^{\prime} / r\right) \tan \alpha$. Thus (A37) becomes

$$
\begin{equation*}
\sin 2 \alpha^{\prime}=\frac{\sin 2 \alpha}{Z \sin \theta}\left(\frac{V_{p}^{\prime}}{V_{p}}\right) \tag{A38}
\end{equation*}
$$

where $\sin 2 \alpha=0.94$.

For $\quad V_{p}^{\prime}=V_{p}, \quad \sin \theta=1 \quad$ and $Z=Z_{\text {max }}$, we obtain from (A38),

$$
\begin{equation*}
\left(\sin 2 \alpha^{\prime}\right)_{\text {min }} \cong 1 / Z_{\text {max }} \cong 10^{-2} \tag{A39}
\end{equation*}
$$

whence we obtain $\left(\sin \alpha^{\prime}\right)_{\text {min }} \cong 10^{-3}$. Thus $\quad\left(\cos \alpha^{\prime}\right)_{\max }=\sqrt{1-\left(\sin ^{2} \alpha^{\prime}\right)_{\text {min }}} \cong 1$. On the other hand, $\left(\sin 2 \alpha^{\prime}\right)_{\max }=1$. Therefore, $\quad\left(\sin \alpha^{\prime}\right)_{\max }=\sin 45^{\circ}=0.707$ and $\quad\left(\cos \alpha^{\prime}\right)_{\min }=\sqrt{1-\left(\sin ^{2} \alpha^{\prime}\right)_{\max }} \cong 0.7$. Thus

$$
0.7<\cos \alpha^{\prime}<1
$$

Substitution of $V_{p}=3.8 \times 10^{7} \mathrm{~m} / \mathrm{s}$, $\cos ^{2} \alpha=0.333$ and $\cos ^{2} \alpha^{\prime} \cong 0.8$ into Eq.(A34) yields
$m_{g p}=\left(1-3.9 \times 10^{-2} Z \sin \theta\right) m_{p}$
The density of electromagnetic energy in an electromagnetic field is $W=\frac{1}{2} \varepsilon E^{2}+\frac{1}{2} \mu H^{2}$. It is known that $E=v B=v \mu H \quad$ where, according to (54), $v=\sqrt{2 \omega / \mu \sigma}$ for $\sigma \gg \omega \varepsilon$. Thus
$W=\frac{1}{2}\left(\frac{2 \omega \varepsilon}{\sigma}+1\right) \mu H^{2} \cong \frac{1}{2} \mu H^{2}$
For $\sigma \ll \omega \varepsilon$, Eq.(54) shows that $v=c / \sqrt{\varepsilon_{n} \mu}$ and therefore we obtain $W=\mu H^{2}$.

In the case of nuclear protons inside $\vec{H}$, the work to bring the dipole from $\varphi=0$ (null energy) to $\varphi$ is

$$
\begin{equation*}
U=\int_{0}^{\varphi} \tau_{p} d \varphi \tag{A42}
\end{equation*}
$$

where $\tau_{p}=M_{J_{p z}} B \sin \varphi$ in agreement with (A27); $B=\mu_{0} H$. Thus we can write $U=\int_{0}^{\frac{\pi}{2}-\theta} \tau_{p} d \varphi=$
$=M_{J p z} B \int_{0}^{\frac{\pi}{2}-\theta} \sin \varphi d \varphi=M_{J p z} B \sin \theta$
This energy is equal to the energy "extracted" from the field to align each nuclear proton at the same direction $(\theta)$, which is given by

$$
\begin{equation*}
U=\left(1 / 2 \mu_{0} H^{2} V_{*}\right) / 2 Z \tag{A44}
\end{equation*}
$$

where $V_{*}$ is the volume of the sphere
of radius $r_{*}$, around the nucleus, defined as follows:
The density of energy $W$ is the same in each point inside $H$. This means that the density of energy in each orbital electron,

$$
W_{e}=M_{\text {Jez }} B \sin \theta / V_{\text {orbitalspace }}
$$

is equal to the density in each proton,

$$
W_{p}=M_{J p z} B \sin \theta / V_{*}
$$

where $V_{\text {orbiatspace }}=4 / 3 \pi\left(r_{\text {atom }}-r_{*}\right)^{3} ; r_{\text {atom }}$ is the radius of the atom, and $V_{*}=4 / 3 \pi r_{*}^{3}$. Thus

$$
\frac{M_{\text {Jez }}}{\left(r_{\text {atom }}-r_{*}\right)^{3}}=\frac{M_{J_{p z}}}{r_{*}^{3}}
$$

whence,
$r_{*}=\frac{r_{\text {atom }}}{1+\sqrt[3]{M_{\text {Jez }} / M_{J p z}}}=0.103 r_{\text {atom }}$
since $\quad M_{J e z}=\gamma_{e}\left(e / 2 m_{e}\right) J_{e z}=9.27 \times 10^{-24} A . m^{2}$ and $M_{J_{p z}}=1.41 \times 10^{-26}$ A. $\mathrm{m}^{2}$.

Note that $r_{*}$ is smaller than the radius of the first electronic orbit of the atom $\left(\sim 5 \times 10^{-11} \mathrm{~m}\right)$.

Comparison of (A43) and (A44) shows that

$$
\begin{align*}
\sin \theta & =\left(\frac{V_{*}}{4 Z \mu_{0} M_{J_{p z}}}\right) B=6.46 \times 10^{28} \frac{r_{\text {atom }}^{3}}{Z} B= \\
& =6.46 \times 10^{28} \frac{r_{\text {atom }}^{3}}{Z}\left(\frac{d \Phi_{B}}{\cos \varsigma d S}\right) \quad(A 46) \tag{A46}
\end{align*}
$$

since

$$
d \Phi_{B}=\vec{B} \cdot d \vec{S}=B \cos \varsigma d S
$$

Note that the expression of $\sin \theta$ as function of $B$ yields only positive values of $\sin \theta$. Thus, it is necessary the substitution of $B$ by $d \Phi_{B} / \cos \varsigma d S$ to show explicitly the negative values of $\sin \theta$. Thus, if $\varsigma>\pi / 2$ the Eq.(A46) yields $\sin \theta<0$. In this case, the sign ( - ) in Eq.(A40) becomes (+).

In practice when a magnet approaches from a metallic sample, a magnetic field is induced at the sample ( in opposition to the magnetic field of
the magnet ). Under these conditions, the angle $\varsigma$ between the induced magnetic flux density $\vec{B}$ and $d \vec{S}$ is zero, and therefore the magnetic flux $\Phi_{B}=\vec{B} . d \vec{S}$ will be positive and, in accordance with (A40), there will be a decrease in the gravitational mass of the sample. However, when the magnet is moved backward the direction of the induced magnetic flux density is inverted ( the angle $\varsigma$ becomes equal to $180^{\circ}$ ) making $\Phi_{B}=\vec{B} . d \vec{S}$ negative. In this case there will be an increase in the gravitational mass of the sample. It was implicitly assumed in both cases that the oscillating period ( $T$ ) of the magnet is relatively long.

If $T$ is short the concept of Effective or Root-Mean-Square Value may be extended to the periodic magnetic flux $\Phi_{B}=\Phi_{B}(t)$. The result is

$$
\begin{equation*}
\Phi_{B(r m s)}=\sqrt{\frac{1}{T} \int_{0}^{T} \Phi_{B}{ }^{2} d t} \tag{A47}
\end{equation*}
$$

Thus, the effective value of $\Phi_{B}$ is $\Phi_{B(r m s)}$ (always positive). This means that, $\cos \varsigma$ will be always positive. Consequently, $\sin \theta$ will be always positive too, and the sign (-) in Eq.(A40) does not change, i.e., it will be always negative.

For $\sin \theta$ always positive we may take the expression of $\sin \theta$ as function of $B$ in (A46). In addition, if the oscillating period of the magnetic flux $\Phi_{B}=\Phi_{B}(t)$ is short we may change $B$ for $B_{r m s}$, and consequently the substitution of (A46) into (A40) yields

$$
m_{g p}=\left\lfloor 1-2.52 \times 10^{27} r_{\text {atom }}^{3} B_{r m s}\right\rfloor m_{p}(A 48)
$$

For a sinusoidal waveform, the effective value of the magnetic flux density, as we know, is

$$
B_{r m s}=\frac{B_{m}}{\sqrt{2}}
$$

According to (A46) the value of $\sin \theta$ will be null when $d \Phi_{B}=0$. Consider then the Maxwell's equation

$$
\begin{equation*}
\nabla \times \vec{E}=-\frac{\partial \vec{B}}{\partial t} \tag{A49}
\end{equation*}
$$

whence we obtain

$$
\begin{align*}
\int_{S}(\nabla \times \vec{E}) d \vec{S} & =-\int_{S} \frac{\partial B}{\partial t} d \vec{S}=-\frac{d}{d t} \int_{S} \vec{B} \cdot d \vec{S}= \\
& =-\frac{d \Phi_{B}}{d t} \tag{A50}
\end{align*}
$$

If $B$ does not depends on time (constant with respect to the time) then $\partial B / \partial t=0 \quad$ and (A49) shows that $\nabla \times \vec{E}=0$. Thus (A50) gives

$$
d \Phi_{B}=0
$$

In this case, Eq.(A46) gives $\sin \theta=0$ and consequently Eq.(A40) reduces to $m_{g p}=m_{p}$.

Now consider Eq.(A48). Note that besides $B_{r m s}$ it depends on $r_{\text {atom }}^{3}$. The values of $r_{\text {atom }}$ for light elements such as Mg , $\mathrm{Al}, \mathrm{Si}$, are $1.75 \times 10^{-10} \mathrm{~m}$ $1.43 \times 10^{-10} \mathrm{~m}, 1.17 \times 10^{-10} \mathrm{~m}$ respectively. For heavy elements such as Pt, Au, Pb , the values are $1.38 \times 10^{-10} \mathrm{~m}$, $1.44 \times 10^{-10} \mathrm{~m}, \quad 1.75 \times 10^{-10} \mathrm{~m}$ respectively. Thus, for Mg or Pb the Eq.(A48) gives

$$
\begin{equation*}
m_{g p}=\left\lfloor 1-1.35 \times 10^{-2} B_{r m s}\right\rfloor m_{p} \tag{A51}
\end{equation*}
$$

which shows that to yield a variation > $1 \%$ in the gravitational mass of the protons of Mg or Pb , it is necessary $B_{r m s}>0.74 T$ Recently, superconducting magnets are able to produce $14.7 T$, therefore the validity of this part of the theory can be checked.

An interesting situation arises when a body penetrates a magnetic field in the direction of its center. According to (A48) the gravitational masses of the protons of the body decrease progressively. This is due to
the intensity increase of the magnetic field upon the body while it penetrates the field. Equation (4) shows that the decreasing of gravitational mass of the body reduces its momentum. This means that for the case of collisions the impact produced by the body will be less than in the absence of the magnetic field.

In order to understand this phenomenon we might, based on (45), think of the inertial mass as being formed by two parts: one positive and another negative. Thus, when the body penetrates the magnetic field its negative inertial mass increase, but its total inertial mass decreases, i.e., although there is an increase of inertial mass, the total inertial mass (which is equivalent to gravitational mass) will be reduced. Consequently the momentum of the body will also be reduced.

A strong magnetic field can therefore function as a protection shielding against the impact of high speed particles. This may be particularly important for the protection of spacecrafts during spatial flights.

## APPENDIX B

Equation (59) shows that the gravitational mass of a supermalloy wire is strongly decreased when the electric current through the wire has extremely-low frequency. As we have seen, for $i_{0}=0.04 A$ and $f \cong 1.7 \times 10^{-6} \mathrm{~Hz}$ at $\quad t=1 / 4 f=1.47 \times 10^{5} s \cong 40.8 h$ Eq.(59) gives $m_{g(m n)} \cong-m_{i(m))}$.

The period of this wave is too long. In order to reduce the period of the wave we can reduce the diameter of the wire. For example, in the case of supermalloy or mumetal wire 0.005" diameter, the period will be strongly reduced down to $\sim 100 \mathrm{~s}$. In addition, by digitizing the top of this ELF wave, as shown in Fig.4, we may produce a

(a)


ELF digitized wave which obviously becomes much more adequate for practical use.

This possibility points to some interesting systems as shown in Fig. 5. Figures 5(a) and 5(b) show the generation of Lift Force. Figure 5(c) shows a new concept of motor : the Gravitational Motor, also based on the gravity control.

When the gravitational field of an object changes, the changes ripple outwards through spacetime. These ripples are called gravitational radiation or gravitational waves.

The existence of gravitational waves follows from the General Theory of Relativity. In Einstein's theory of gravity the gravitational waves propagate at the speed of light.

Just as electromagnetic waves (EM), gravitational waves (GW) too carry energy and momentum from their sources. Unlike EM waves, however, there is no dipole radiation in Einstein's theory of gravity. The dominant channel of emission is quadrupolar. But the existence of negative gravitational mass suggest the possibility of dipole gravitational radiation.

This fact is highly relevant because now we can build a gravitational wave transmitter to generate detectable levels of gravitational radiation. Gravitational waves are very suitable as a means of transmitting information because of their low interaction an therefore low scattering. In figure 5(d) we present the Gravitational Radiation Transmitter, a new concept of transmitter that arises from this new technology.

Fig. 4


Fig. 5 - Schematic diagram of systems using gravity control:
(a) and (b) The generation of Gravitational Lift Force. (c) The Gravitational Motor
(d) The Gravitational Radiation Transmitter.

## APPENDIX C

In this appendix we will show that strong fluxes of ELF radiation upon electric/electronic circuits can suddenly increase the electric currents and consequently to damage these circuits.

Let us consider an electric current $I$ through a conductor subjected to electromagnetic radiation with power density $D$ and frequency $f$.

Under these circumstances the gravitational mass $m_{g e}$ of the electrons of the conductor, according to Eq. (58), is given by

$$
\begin{equation*}
m_{g e}=\left\{1-2\left[\sqrt{1+\left(\frac{\mu \sigma D}{4 \pi f \rho c}\right)^{2}}-1\right]\right\} m_{e} \tag{C1}
\end{equation*}
$$

where $m_{e}=9.11 \times 10^{-31} \mathrm{~kg}$.
Note that $m_{g e}$, becomes less than the inertial mass, $m_{e}$. If the radiation upon the conductor has extremely-low frequency (ELF radiation) then $m_{g e}$ can be strongly reduced. For example, if $f \approx 10^{-6} \mathrm{~Hz}$, $D \approx 10^{5} \mathrm{~W} / \mathrm{m}^{2}$ and the conductor is made of copper ( $\mu \cong \mu_{0} ; \sigma=5.8 \times 10^{7} \mathrm{~S} / \mathrm{m}$ and $\left.\rho=8900 \mathrm{~kg} / \mathrm{m}^{3}\right)$ then

$$
\left(\frac{\mu \sigma D}{4 \pi f \rho c}\right) \approx 1
$$

and consequently $m_{g e} \approx 0.1 m_{e}$.
According to Eq.(6) the force upon each free electron is given by

$$
\begin{equation*}
\vec{F}_{e}=\frac{m_{g e}}{\left(1-V^{2} / c^{2}\right)^{\frac{3}{2}}} \frac{d \vec{V}}{d t}=e \vec{E} \tag{C2}
\end{equation*}
$$

where $E$ is the applied electric field. Therefore the decreasing of $m_{g e}$ produces an increase in the velocity $V$ of the free electrons and consequently the drift velocity $V_{d}$ is also increased. It is known that the density of electric current $J$ through a conductor [24] is given by

$$
\begin{equation*}
\vec{J}=\Delta_{e} \vec{V}_{d} \tag{C3}
\end{equation*}
$$

where $\Delta_{e}$ is the density of the free
electric charges ( For cooper conductors $\quad \Delta_{e}=1.3 \times 10^{10} \mathrm{C} / \mathrm{m}^{3} \quad$ ). Therefore increasing $V_{d}$ produces an increase in the electric current $I$. Thus if $m_{g e}$ is reduced 10 times ( $m_{g e} \approx 0.1 m_{e}$ ) the drift velocity $V_{d}$ is increased 10 times as well as the electric current. This sudden increase in the electric currents of electric/electronic circuits can cause damage.

In order for the ELF radiation to arrive at each electron, the flux density $D$ must be greater than $D_{\text {min }}$ given by

$$
\begin{equation*}
D_{\text {min }}=\frac{h f^{2}}{A_{\text {electron }}} \tag{C4}
\end{equation*}
$$

where $A_{\text {electron }}$ is the "area of cross section" of the electron. We know that the leptons should have length scale less than $10^{-19} \mathrm{~m}$ [25]. This means that an electron has a maximum, "radius" of $r_{e} \sim 10^{-19} \mathrm{~m}$. The plausible relation given by Brodsky and Drell [26] for the simplest composite theoretical model of the electrons, $|g-2|=r_{e} / \lambda_{c}$ or $\quad\left|g-g_{\text {DIRAd }}\right|=r_{e} / \lambda_{c}$, where $\lambda_{c}=3.9 \times 10^{-13} \mathrm{~m}$ and $|g-2|=1.1 \times 10^{-10} \mathrm{~m}$ [27] gives an electron radius of

$$
r_{e} \approx 10^{-22} \mathrm{~m}
$$

Therefore assuming $A_{\text {electron }} \approx 10^{-45} \mathrm{~m}$ (C4) gives

$$
\begin{equation*}
D_{\min } \approx 10^{12} f^{2} \tag{C5}
\end{equation*}
$$

Thus, for $f \approx 10^{-6} \mathrm{~Hz}$ we have $D_{\text {min }} \approx 1 W / \mathrm{m}^{2}$.

Since the orbital electrons moment of inertia is given by $I_{i}=\Sigma\left(m_{i}\right)_{j} r_{j}^{2}$, where $m_{i}$ refers to inertial mass and not to gravitational mass, then the momentum $L=I_{i} \omega$ of the conductor orbital electrons are not affected by the ELF radiation. Consequently this radiation just affects the conductor free electron velocities.

## APPENDIX D

Here we will show that the possible existence of ELF radiation into solar radiation can explain the anomalous acceleration which has been observed on the Pioneer 10 and 11 spacecrafts in the solar system [28] and also the anomalous behavior of mechanical systems during solar eclipses observed by Allais [29] with paraconical pendula and Saxl and Allen [30] with a torsion pendulum and measurements with gravimeters.

Equation (58) shows that the presence of ELF radiation (frequency ranging between $0.1 \mu \mathrm{~Hz}$ down to 0.1 nHz ) into solar radiation can slightly reduce the gravitational masses of any body in the solar system. The gravitational mass of these bodies become less than their inertial masses, $m_{i}$, as expressed by
$m_{g}=\left\{1-2\left[\sqrt{1+\left(\frac{\mu \sigma D}{4 \pi f \rho c}\right)^{2}}-1\right]\right\} m \quad(D 1)$
The total energy of the spacecraft (Hamiltonian) according to (20), is $H=\sqrt{p^{2} c^{2}+m_{g}^{2} c^{2}}$. Therefore the decreasing of $m_{g}$ reduces the total energy of the spacecraft, and consequently its acceleration. This explains the fact that the Pioneer 10 and 11 spacecrafts, launched by NASA, in the early 1970s, are receding from the sun slightly more slowly than they should be.

Similarly, the ELF solar radiation slightly reduces the gravitational mass of the Earth, $M_{g \oplus}$, and consequently it becomes smaller than its inertial mass $M_{i \oplus}$.

From Electrodynamics we know that radiation with frequency $f$ propagating within a material with electromagnetic characteristics $\varepsilon, \mu$ and $\sigma$ has the amplitudes of its waves
attenuated by $\mathrm{e}^{-1}=0.37$ (37\%) when it penetrates a distance $z$, given by

$$
\begin{equation*}
z=\frac{1}{\omega \sqrt{1 / 2 \varepsilon \mu\left(\sqrt{1+(\sigma / \omega \varepsilon)^{2}}-1\right)}} \tag{D2}
\end{equation*}
$$

The radiation is mostly absorbed if it penetrates a distance $\delta \cong 5 z$.

Based on this equation we can easily conclude that the ELF solar radiation is mostly absorbed by the moon, therefore during the eclipses, when the moon passes in front of the sun, the ELF solar radiation ceases to fall upon the Earth and, according to (D1), the gravitational mass of the Earth increases ( $M_{g \oplus}$ becomes equal to $M_{i \oplus}$ ) slightly increasing the gravity $g=G M_{g \oplus} / r^{2}$. Similarly, the gravitational mass of the pendulum, $m_{g}$, also increases slightly during the eclipse. Since the period, $T$, of the pendulum is given by

$$
\begin{equation*}
T=2 \pi \sqrt{\left(\frac{m_{i}}{m_{g}}\right) \frac{l}{g}} \tag{D3}
\end{equation*}
$$

one can conclude that during the eclipses the pendulum's periods are slightly decreased. This means that their motion becomes faster during the eclipses, such as has been observed in the experiments of Allais, Saxl and Allen.

## APPENDIX E

Equation (70) shows that the gravitational interaction can be repulsive, besides attractive. Therefore, as with electromagnetic interaction, the gravitational interaction must be produced by the exchange of "virtual" quanta of spin 1 and mass null, i.e., the gravitational "virtual" quanta (gravitons) must have spin 1 and not 2.

It is known that the gravitational interaction is instantaneously communicated to all the particles of the Universe. This means that the velocity of the gravitational "virtual" quanta must be infinite.

Consider a Mumetal ELF antenna as showed in Fig.6. The ELF electric current through it is $i_{e}=i_{0} \sin \omega t=i_{0} \sin 2 \pi f t$. According to (59) the gravitational mass of the antenna is given by

$$
m_{g}=\left\{1-2\left[\sqrt{1+\left(i_{e}^{4} \mu / 64 \pi^{3} c^{2} \rho^{2} S^{4} f^{3} \sigma\right)}-1\right]\right\} m_{i}(E 1)
$$

where $\rho, \mu, \sigma$ and $S$ are respectively the density, the magnetic permeability, the electric conductivity and the area of the antenna cross section.

It is easy to see that the ELF electric current yields a variation in the gravitational mass of the antenna, which is detected instantaneously by all particles of the Universe, i.e., the gravitational "virtual" quanta emitted from the antenna will instantaneously reach all particles.

When a particle absorbs photons, the momentum of each photon is transferred to particle and, in accordance with (41), the gravitational mass of the particle is altered. Similarly to the photons the gravitational "virtual" quanta have mass null and momentum. Therefore the gravitational masses of the
particles are also altered by the absorption of gravitational "virtual" quanta.

If the gravitational "virtual" quanta are emitted by an antenna (like a Mumetal ELF antenna) and absorbed by a similar antenna, tunned to the same frequency $f$, the changes on the gravitational mass of the receiving antenna, in accordance with the principle of resonance, will be similar to changes occurred on the transmitting antenna, and consequently the induced current through the receiving antenna has the same frequency $f$ and, in agreement with (E1), must be similar to electric current through the transmitting antenna. The Fig. 7 shows the emission and detection of gravitational "virtual" quanta by two Mumetal ELF antennas.

Note that the changes of gravitational mass of the antenna also produce the so-called gravitational waves which are ripples in the geometry of the spacetime. This is produced by the changes on the gravitational field of the antenna. When the gravitational field changes, the changes ripple outwards through space and take a finite time to reach other objects. In Einstein's theory of gravity these ripples (gravitational waves) propagate at the speed of light ( $c$ ).

Therefore the velocity of the gravitational waves is much less than the velocity $(\infty)$ of the gravitational "virtual" quanta (gravitons). There is another fundamental difference between the gravitational waves and gravitons: the gravitational waves are real unlike the gravitons which are virtual.

Note that a Mumetal ELF antenna emits gravitons and gravitational waves simultaneously. Thus it is not only a gravitational


$$
\mathrm{V}=c
$$

Gravitational Waves

$$
\mathrm{V}=c
$$

Fig. 6


Transmitter
Fig. 7
antenna: it is a macroscopic quantum gravitational antenna because can also emit and detect gravitational "virtual" quanta, which can to transmit information instantaneously from any distance in the Universe without scattering.

Unlike the electromagnetic waves the gravitational waves have low interaction and consequently low scattering. Therefore gravitational waves are suitable as a means of transmitting information. However, when the distance between transmitter and receiver is too large, for example of the order of magnitude of several light-years, the transmission of information by means of gravitational waves becomes impracticable due to the long time necessary to receive the information. The velocity of the gravitational waves is equal to the speed of light $(c)$ therefore the delay would be in the order of several years.

The velocity of the gravitational "virtual " quanta is infinite thus there is no delay during the transmissions. The scattering of this radiation is null. Therefore this gravitational "virtual" radiation or gravitational "virtual" waves are very suitable as a means of transmitting information at any distances including astronomical distances.

In order to check these theoretical predictions we propose the following experiment: A transmitter and a receiver both with Mumetal antennas will be placed in two very distant places, like Mars and Earth (the distance is $\sim 7.9 \times 10^{10} \mathrm{~m}$ ). Electromagnetic waves or gravitational real waves emitted from Mars will need ~ 4.4 minutes to arrive at Earth. There is no delay in the case of gravitational "virtual" waves due to their infinite velocity. Therefore simply checking that there is no delay during the transmission by using Mumetal antennas we can check the existence of the gravitons.

Since the gravitational masses of the antennas vary during the transmissions then another way to check the existence of the gravitons is to measure the weight of the receiving and transmitting antennas during the transmissions. In this case is not necessary to put the antennas in very distant places.

It is easy to see that the information transportation with infinite velocity by means of gravitons promises to be quite useful for the Internet (Quantum Internet) and also for the development of Quantum Teleportation Systems.

By operating with infinite velocity and not with the speed of light these systems will solve in the future the problem of the cosmic transportation of long range, since it is impracticable for spacecrafts - even with velocities very close to light speed - to reach places whose distances are greater than 100 light-years.

## REFERENCES

[1] Isham, C. J. (1975) Quantum gravity, an Oxford Symposium, OUP.
[2] Isham, C.J., (1997) "Structural Problems Facing Quantum Gravity Theory", in M, Francaviglia, G, Longhi, L, Lusanna, and E, Sorace, eds., Proceedings of the 14th International Conference on General Relativity and Gravitation, 167-209, (World Scientific, Singapore, 1997).
[3] Landau, L. and Lifchitz, E. (1969) Theorie du Champ, Ed.MIR, Moscow, Portuguese version (1974) Ed. Hemus, S.Paulo, p. 35.
[4] Landau, L. and Lifchitz, E. (1969) Mecanique, Ed.MIR, Moscow, 15.
[5] Landau, L. and Lifchitz, E.[3], p. 35.
[6] Landau, L. and Lifchitz, E.[3], p.36.
[7] Beiser, A. (1967) Concepts of Modern Physics, McGraw-Hill, Portuguese version (1969) Ed. Poligno, S.Paulo, p. 151.
[8] Schiff, L.I. (1981) Quantum Mechanics, McGraw-Hill, p. 54.
[9] Landau, L. and Lifchitz, E.[3], p. 324.
[10] Landau, L. and Lifchitz, E.[3], p. 357.
[11] Landau, L.and Lifchitz,E.[3],p.p.358-359.
[12] Landau, L. and Lifchitz, E.[3], p. 64.
[13] Luminet, J. at al., (2003) Nature 425, 593-595.
[14] Ellis, G.F.R (2003) Nature 425, 566-567.
[15] Landau, L. and Lifchitz, E.[3], p.440-441.
[16] Landau, L. and Lifchitz, E.[3], p. 333.
[17] Landau, L. and Lifchitz, E.[3], p. 331.
[18] Landau, L. and Lifchitz, E.[3], p. 327.
[19] Hawking, S.W (1971) MNRAS., 152, 75.
[20] Carr, B.J.(1976) Astrophys.J., 206,10.
[21] Beiser, A. (1967) Concepts of Modern Physics, McGraw-Hill, Portuguese version (1969) Ed. Poligno, S.Paulo, p.223.
[22] Alonso, M.and Finn, E.(1967)Fundamental University Physics,Addison-Wesley,Portuguese version (1972) Ed. Blücher, S.Paulo, p.79-80.
[23] Alonso, M.and Finn, E.(1967)Fundamental University Physics,Addison-Wesley,Portuguese version (1972) Ed. Blücher, S.Paulo, p. 89.
[24] Quevedo, C.P.(1978) Eletromagnetismo, McGraw-Hill, p. 102.
[25] Fritzsch, H. (1984) Quarks-Urstoff unserer Welt, R. Piper GmbH\&Co. KG,München, Portuguese version (1990), Ed. Presença, Lisboa, p. 215.
[26] Brodsky, S . J., and Drell, S . D., (1980) Anomalous Magnetic Moment and Limits on Fermion Substructure, Phys. Rev. D, 22, 2236.
[27] Dehmelt, H.G.,(1989) Experiments with an isolated subatomic particle at rest, Nobel Lecture, p. 590.
[28] Anderson, J.D, Lang, P.A, Lau, E. L, Liu, A.S, Nieto, N.M and Turyshev, S.G. (2002) Study of the anomalous acceleration of Pioneer 10 and 11, Phys. Rev. D 65, 082004. gr-qc/0104064.
[29] Allais, M.F.C (1957) Mouvement $d u$ pendule paraconique et éclipse totale de Soleil 30 jui 1954, C.R. Acad. Sci, 245,2001.
[30] Saxl, E.J. and Allen, M.(1971) 1970Solar Eclipse as "Seen" by a Torsion Pendulum, Phys.rev. D3, 823.

